Chapter 06 이산 시간 푸리에 급수/변환

[Quick Review]

(1) ×
(2) 주기
(3) ×
(4) 단위원, 등간격
(5) ○
(6) ×
(7) 반주기
(8) 연속, 주기
(9) ×
(10) ×
(11) 에너지
(12) 전력
(13) 같다
(14) 주기⇔이산
(15) ○
(16) ○
(17) 실수
(18) 원형
(19) ×
(20) 단위원

[기초 문제]

6.1 ㉯

6.2 ㉱

6.3 ㉮

6.4 ㉱

6.5 ㉰

6.6

(a) $X_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n} = \frac{1}{4}(1+2e^{-j\frac{\pi}{2}k}+e^{-j\pi k})$

$$X_0 = \frac{1}{4}(1+2+1) = 1$$

$$X_1 = \frac{1}{4}(1+2e^{-j\frac{\pi}{2}}+e^{-j\pi}) = -j\frac{1}{2}$$

$$X_2 = \frac{1}{4}(1+2e^{-j\pi}+e^{-j2\pi}) = 0$$

$$X_3 = \frac{1}{4}(1+2e^{-j\frac{3\pi}{2}}+e^{-j3\pi}) = j\frac{1}{2}$$

(b) $X_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n} = \frac{1}{4}(-1+2e^{-j\frac{\pi}{2}k}-e^{-j\pi k})$

$$X_0 = \frac{1}{4}(-1+2-1) = 0$$

$$X_1 = \frac{1}{4}(-1+2e^{-j\frac{\pi}{2}}-e^{-j\pi}) = -j\frac{1}{2}$$

$$X_2 = \frac{1}{4}(-1+2e^{-j\pi}-e^{-j2\pi}) = -1$$

$$X_3 = \frac{1}{4}(-1+2e^{-j\frac{3\pi}{2}}-e^{-j3\pi}) = j\frac{1}{2}$$

(c) $X_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n} = \frac{1}{4}(1+e^{-j\frac{\pi}{2}k})$

$$X_0 = \frac{1}{4}(1+1) = \frac{1}{2}$$

$$X_1 = \frac{1}{4}(1+e^{-j\frac{\pi}{2}}) = \frac{1}{4}-j\frac{1}{4}$$

$$X_2 = \frac{1}{4}(1+e^{-j\pi}) = 0$$

$$X_3 = \frac{1}{4}(1+e^{-j\frac{3\pi}{2}}) = \frac{1}{4}+j\frac{1}{4}$$

(d) $X_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n} = \frac{1}{4}(e^{-j\pi k} + e^{-j\frac{3\pi}{2}k})$

$$X_0 = \frac{1}{4}(1+1) = \frac{1}{2}$$

$$X_1 = \frac{1}{4}(e^{-j\pi} + e^{-j\frac{3\pi}{2}}) = -\frac{1}{4} + j\frac{1}{4}$$

$$X_2 = \frac{1}{4}(e^{-j2\pi} + e^{-j3\pi}) = 0$$

$$X_3 = \frac{1}{4}(e^{-j3\pi} + e^{-j\frac{9\pi}{2}}) = -\frac{1}{4} - j\frac{1}{4}$$

6.7

(a) $X_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n} = \frac{1}{2}$

$$X_0 = X_1 = \frac{1}{2}$$

(b) $X_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n} = \frac{1}{2}(1 - e^{-jk\pi})$

$$X_0 = \frac{1}{2}(1 - e^{-j0\pi}) = 0$$

$$X_1 = \frac{1}{2}(1 - e^{-j\pi}) = 1$$

(c) $X_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\Omega_0 n} = \frac{1}{N}, \quad \Omega_0 = \frac{2\pi}{N}$

(d) $X_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n} = \frac{1}{N}(1 + 2\cos(k\Omega_0)), \quad \Omega_0 = \frac{2\pi}{N}$

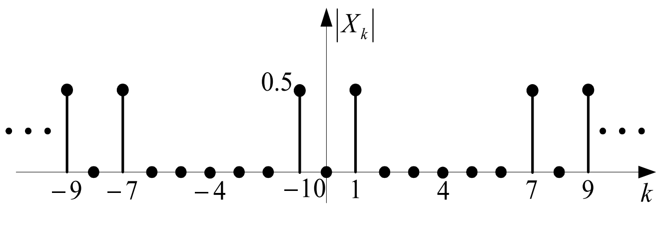
6.8

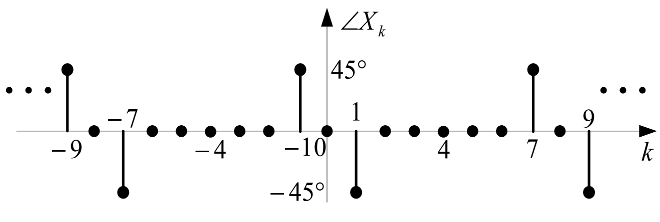
(a) $X_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk\Omega_0 n} = \frac{1}{8}\sum_{n=0}^{7} x[n]e^{-jk\frac{\pi}{4}n}$

$$X_1 = \frac{1}{2}e^{-j\frac{\pi}{4}} = \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}$$

$$X_7 = \frac{1}{2}e^{j\frac{\pi}{4}} = \frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}$$

$$X_k = 0, \quad k = 0,\ 2,\ 3,\ 4,\ 5,\ 6$$





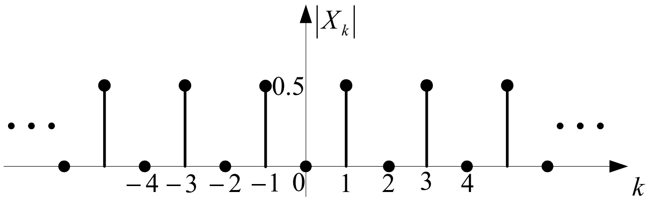
(b) $X_k = \dfrac{1}{N}\sum_{n=<N>} x[n]\,e^{-jk\Omega_0 n} = \dfrac{1}{4}\sum_{n=0}^{3} x[n]\,e^{-jk\frac{3\pi}{2}n}$

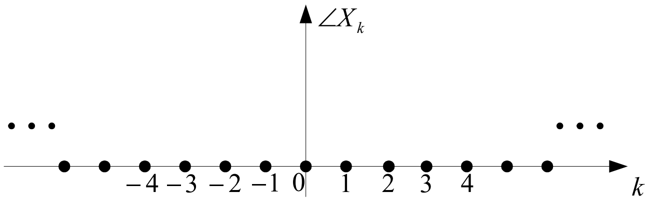
$$X_0 = 0$$

$$X_1 = \frac{1}{2}$$

$$X_2 = 0$$

$$X_3 = \frac{1}{2}$$





(c) $X_k = \dfrac{1}{N}\sum_{n=<N>} x[n]\,e^{-jk\Omega_0 n} = \dfrac{1}{12}\sum_{n=0}^{11} x[n]\,e^{-jk\frac{\pi}{6}n}$

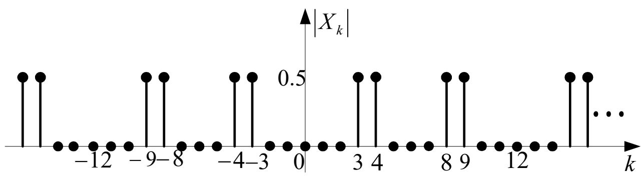
$$X_3 = \frac{1}{2}e^{-j\frac{\pi}{2}}$$

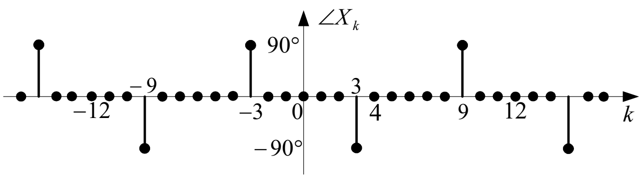
$$X_4 = \frac{1}{2}$$

$$X_8 = \frac{1}{2}$$

$$X_9 = \frac{1}{2}e^{j\frac{\pi}{2}}$$

$$X_k = 0, \quad k = 0,\ 1,\ \cdots,\ 11, \quad k \neq 3,\ 4,\ 8,\ 9$$





6.9

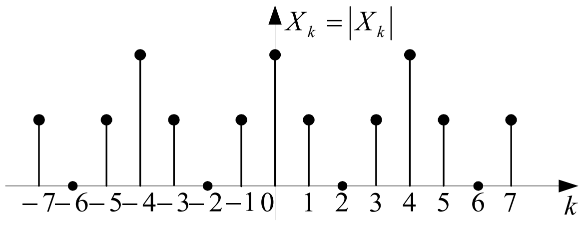
(a) $X_0 = \frac{1}{2}\left(1+\cos\frac{0\pi}{2}\right) = 1 \qquad X_2 = \frac{1}{2}\left(1+\cos\frac{2\pi}{2}\right) = 0$

$X_1 = \frac{1}{2}\left(1+\cos\frac{\pi}{2}\right) = \frac{1}{2} \qquad X_3 = \frac{1}{2}\left(1+\cos\frac{3\pi}{2}\right) = \frac{1}{2}$

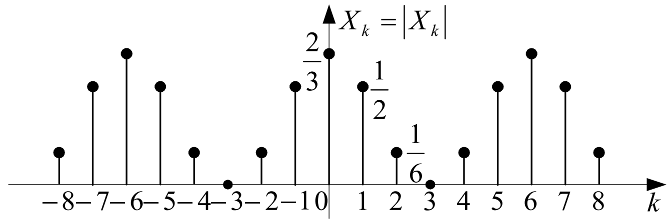
(b) $X_0 = \frac{1}{3}\left(1+\cos\frac{0\pi}{3}\right) = \frac{2}{3} \qquad X_3 = \frac{1}{3}\left(1+\cos\frac{3\pi}{3}\right) = 0$

$X_1 = \frac{1}{3}\left(1+\cos\frac{\pi}{3}\right) = \frac{1}{2} \qquad X_4 = \frac{1}{3}\left(1+\cos\frac{4\pi}{3}\right) = \frac{1}{6}$

$X_2 = \frac{1}{3}\left(1+\cos\frac{2\pi}{3}\right) = \frac{1}{6} \qquad X_5 = \frac{1}{3}\left(1+\cos\frac{5\pi}{3}\right) = \frac{1}{2}$



(a)



(b)

6.10

(a) $x[n] = \sum_{k=-4}^{3} X_k e^{jk\frac{\pi}{4}n} = 2 + 2\cos(\frac{3\pi}{4}n)$

(b) $x[n] = \sum_{k=0}^{7} X_k e^{jk\frac{\pi}{4}n} = (1+(-1)^n)(1-2\cos(\frac{\pi}{4}n)) + 2\cos(\frac{\pi}{2}n)$

$= \begin{cases} 8, & n = 4,\ 12,\ 20,\ 28, \cdots, \\ 0, & \text{그 외} \end{cases}$

$= [\ \cdots\ \check{0},\ 0,\ 0,\ 0,\ 8,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 8,\ 0,\ 0,\ 0,\ 0,\ \cdots\ ]$

(c) $x[n] = \sum_{k=0}^{7} X_k e^{jk\frac{\pi}{4}n} = 1 + (-1)^n(1 + 2\cos(\frac{\pi}{2}n))$

$x[0] = 4$
$x[1] = 0$
$x[2] = 0$
$x[3] = 0$
$x[4] = 4$
$x[5] = 0$
$x[6] = 0$
$x[7] = 0$

(d) $x[n]=\sum_{k=0}^{7} X_k e^{jk\frac{\pi}{4}n} = 4(-1)^n \cos(\frac{\pi}{4}n)\cos(\frac{\pi}{2}n)$

$x[0]=4$
$x[1]=0$
$x[2]=0$
$x[3]=0$
$x[4]=-4$
$x[5]=0$
$x[6]=0$
$x[7]=0$

6.11

(a) $P = X_0 + \sum_{k=1}^{10} 2|X_k|^2 = 2(\frac{1}{2})^2 + 2(\frac{1}{2})^2 = 1$

(b) $P = X_0 + \sum_{k=1}^{12} 2|X_k|^2 = 1^2 + 2(1^2+2^2+2^2+1^2) = 21$

6.12

(a) $X(\Omega) = \sum_{n=0}^{3} x[n]e^{-j\Omega n} = 2(e^{-j\Omega}+e^{-j2\Omega})\cos\Omega = 4e^{-j\frac{3}{2}\Omega}\cos(\frac{1}{2}\Omega)\cos(\Omega)$

(b) $X(\Omega) = \sum_{n=-2}^{2} x[n]e^{-j\Omega n} = 4\cos(\Omega)+8\cos(2\Omega)$

(c) $X(\Omega) = \sum_{n=1}^{5} x[n]e^{-j\Omega n} = e^{-j3\Omega}(3+4\cos(\Omega)+2\cos(2\Omega))$

(d) $X(\Omega) = \sum_{n=-3}^{3} x[n]e^{-j\Omega n} = -j[6\sin(\Omega)+12\sin(2\Omega)+18\sin(3\Omega)]$

6.13

(a) $X(\Omega) = \frac{2\pi}{N}\sum_{k=-\infty}^{\infty} X'(k\Omega_0)\delta(\Omega-k\Omega_0) = \sum_{k=-\infty}^{\infty} \pi\delta(\Omega-k\pi)$

(b) $X(\Omega) = \frac{2\pi}{N}\sum_{k=-\infty}^{\infty} X'(k\Omega_0)\delta(\Omega-k\Omega_0) = \sum_{k=-\infty}^{\infty} \frac{\pi}{2}(1+e^{-j\frac{\pi}{2}k})\delta(\Omega-\frac{\pi}{2}k)$

(c) $X(\Omega) = \frac{2\pi}{N}\sum_{k=-\infty}^{\infty} X'(k\Omega_0)\delta(\Omega-k\Omega_0) = \sum_{k=-\infty}^{\infty} \pi e^{-j\frac{3\pi}{8}k}(\cos\frac{\pi}{8}k)(\cos\frac{\pi}{4}k)\delta(\Omega-\frac{\pi}{4}k)$

(d) $X(\Omega) = 2\pi\sum_{k=-\infty}^{\infty} X_k\delta(\Omega-k\Omega_0) = \sum_{k=-\infty}^{\infty} 2\pi X_k\delta(\Omega-\frac{\pi}{4}k)$

6.14

(a) $X(\Omega)=\sum_{n=-N}^{N} n e^{-j\Omega n} = -j2\sum_{k=1}^{N} k\sin(k\Omega)$

(b) $X(\Omega) = j\dfrac{dX'(\Omega)}{d\Omega} = j\dfrac{-j0.5e^{-j\Omega}}{(1-0.5e^{-j\Omega})^2} = \dfrac{0.5e^{-j\Omega}}{(1-0.5e^{-j\Omega})^2}$

(c) $X(\Omega)=X_1(\Omega)+X_2(\Omega)=\dfrac{1}{1-0.5e^{-j\Omega}}+\dfrac{1}{1+0.5e^{-j\Omega}}=\dfrac{2}{1-0.25e^{-j2\Omega}}$

(d) $X(\Omega)=X_1(\Omega)+X_2(\Omega)=\dfrac{1}{1-0.5e^{-j\Omega}}+\dfrac{0.5e^{j\Omega}}{1-0.5e^{j\Omega}}=\dfrac{0.75}{1.25-\cos\Omega}=\dfrac{3}{5-4\cos\Omega}$

(e) $X(\Omega)=\dfrac{1}{2}X_1(\Omega-\dfrac{\pi}{2})+\dfrac{1}{2}X_1(\Omega+\dfrac{\pi}{2})=\dfrac{1}{1+(0.5)^2e^{-j2\Omega}}$

(f) $X(\Omega)=\dfrac{1}{2}X_1(\Omega-\dfrac{\pi}{2})+\dfrac{1}{2}X_1(\Omega+\dfrac{\pi}{2})=\dfrac{1}{1-(\dfrac{4}{5}\sin(\Omega))^2}$

6.15

(a) $x[n]=\delta[n]-2\delta[n-3]+4\delta[n+2]+3\delta[n-6]$

(b) $x[n]=\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{-\frac{3\pi}{4}} e^{j\Omega n}\,d\Omega+\dfrac{1}{2\pi}\int_{\frac{3\pi}{4}}^{\pi} e^{j\Omega n}\,d\Omega=\mathrm{sinc}(\pi n)-\dfrac{3}{4}\mathrm{sinc}\left(\dfrac{3\pi}{4}n\right)$

(c) $$\begin{aligned} x[n] &= \frac{1}{2\pi}\int_{-\frac{2\pi}{3}}^{-\frac{\pi}{3}} e^{j2\Omega}e^{j\Omega n}\,d\Omega+\frac{1}{2\pi}\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} e^{j2\Omega}e^{j\Omega n}\,d\Omega \\ &= \frac{1}{\pi(n+2)}\left(\sin\left(\frac{2\pi}{3}(n+2)\right)-\sin\left(\frac{\pi}{3}(n+2)\right)\right) \\ &= \frac{2}{3}\mathrm{sinc}\left(\frac{2\pi}{3}(n+2)\right)-\frac{1}{3}\mathrm{sinc}\left(\frac{\pi}{3}(n+2)\right) \end{aligned}$$

(d) $x[n]=\dfrac{1}{2\pi}\displaystyle\sum_{k=-2}^{2}(-1)^k e^{j\frac{k\pi}{2}n}=\dfrac{1}{\pi}\left(1-2\cos\left(\dfrac{\pi}{2}n\right)+2\cos(\pi n)\right)$

6.16 1

6.17

(a) $H(\Omega) = \dfrac{1-0.5e^{-j\Omega}}{1+0.8e^{-j\Omega}}$

$h[n]=(0.8)^n u[n]-0.5(0.8)^{n-1}u[n-1] = \delta[n]+0.3(0.8)^{n-1}u[n-1]$

(b) $H(\Omega) = \dfrac{e^{-j\Omega}}{1+0.64e^{-j2\Omega}}$

$$\begin{aligned} h[n] &= \frac{1}{2}\left((0.8e^{-j\frac{\pi}{2}})^{(n-1)}u[n-1]+(0.8e^{j\frac{\pi}{2}})^{(n-1)}u[n-1]\right) \\ &= (0.8)^{n-1}\cos(\frac{\pi}{2}(n-1))u[n-1] \end{aligned}$$

(c) $H(\Omega) = \dfrac{e^{-j\Omega}}{1 - \frac{5}{6}e^{-j\Omega} + \frac{1}{6}e^{-j2\Omega}}$

$$h[n] = 6(\frac{1}{2})^n u[n] - 6(\frac{1}{3})^n u[n]$$

(d) $H(\Omega) = \dfrac{2e^{-j\Omega} - e^{-j2\Omega}}{1 - 0.3e^{-j\Omega} - 0.4e^{-j2\Omega}}$

$$= e^{-j\Omega}\left(\frac{10/13}{1+0.5e^{-j\Omega}} + \frac{16/13}{1-0.8e^{-j\Omega}}\right) - e^{-j2\Omega}\left(\frac{5/13}{1+0.5e^{-j\Omega}} + \frac{8/13}{1-0.8e^{-j\Omega}}\right)$$

$$h[n] = \frac{10}{13}(-0.5)^{n-1}u[n-1] + \frac{16}{13}(0.8)^{n-1}u[n-1] - \frac{5}{13}(-0.5)^{n-2}u[n-2] - \frac{8}{13}(0.8)^{n-2}u[n-2]$$

$$= 2\delta[n-1] - \frac{10}{13}(-0.5)^{n-2}u[n-2] + \frac{24}{65}(0.8)^{n-2}u[n-2]$$

6.18

(a) $H(\Omega) = \dfrac{Y(\Omega)}{X(\Omega)} = \dfrac{1 - 2e^{-j\Omega} - e^{-j2\Omega}}{1 + 0.5e^{-j\Omega}}$

(b) $h[n] = (-0.5)^n u[n] - 2(-0.5)^{n-1}u[n-1] - (-0.5)^{n-2}u[n-2]$
$= \delta[n] - 2.5\delta[n-1] + (-0.5)^n u[n-2]$

(c) $y[n] + 0.5y[n-1] = x[n] - 2x[n-1] - x[n-2]$

6.19

(a) $y[n] = \cos(\frac{3\pi}{4}(n+1)) + \cos(\frac{3\pi}{4}n) + \cos(\frac{3\pi}{4}(n-1))$

(b) $y[n] = -\cos(\frac{3\pi}{4}(n+1)) + \cos(\frac{3\pi}{4}n) - \cos(\frac{3\pi}{4}(n-1))$

6.20

(a) $H(\Omega) = \dfrac{1}{1 + 0.5e^{-j\Omega}}$

(b) $h[n] = (-0.5)^n u[n]$

(c) $y[n] = 0.5(-0.5)^n u[n] + 0.5(0.5)^n u[n] = ((-0.5)^{n-1} + (0.5)^{n+1})u[n]$

(d) $y[n] = (-0.5)^n u[n] + 2(-0.5)^{n-3}u[n-3]$

[응용 문제]

6.21

(a) $X'_k = X_k - e^{-j\frac{2\pi}{N}k} X_k = (1 - e^{-j\frac{2\pi}{N}k})X_k$

(b) $Y_k = X_k + X_{-k}$

(c) $X'_k = \frac{1}{N}\sum_{n=<N>} x[n]e^{-j\frac{2\pi}{N}(k-\frac{N}{2})n} = X_{k-\frac{N}{2}}$

(d) $X'_k = (1 + e^{j\pi k})X_k = (1 + (-1)^k)X_k$

6.22

(a) $Y(\Omega) = X(-\Omega) = \frac{1}{1 - 0.5e^{j\Omega}}$

(b) $Y(\Omega) = e^{-j\Omega}X(\Omega) = \frac{e^{-j\Omega}}{1 - 0.5\,e^{j\Omega}}$

(c) $Y(\Omega) = j\frac{dX(\Omega)}{d\Omega} = -\frac{0.5e^{-j\Omega}}{(1 + 0.5e^{-j\Omega})^2}$

(d) $Y(\Omega) = e^{j\Omega}X(\Omega) + e^{-j\Omega}X(\Omega) = \frac{2\cos(\Omega)}{1 - 0.5\,e^{-j\Omega}}$

(e) $Y(\Omega) = \frac{1}{2\pi}(\frac{1}{1 - 0.5e^{-j\Omega}}) * \pi(\delta(\Omega + \pi) + \delta(\Omega - \pi)) = \frac{1}{1 + 0.5\,e^{-j\Omega}}$

(f) $Y(\Omega) = X(\Omega)X(\Omega) = \frac{1}{(1 - 0.5e^{-j\Omega})^2}$

6.23

(a) $Re\{X(\Omega)\} = X_e(\Omega) = \sum_{n=-\infty}^{\infty}\frac{x[n] + x[-n]}{2}e^{-j\Omega n}$

$$X_2(\Omega) = Re\{X_1(\Omega + \frac{2\pi}{3})\} + Re\{X_1(\Omega)\} + Re\{X_1(\Omega - \frac{2\pi}{3})\}$$
$$= \sum_{n=-\infty}^{\infty}\frac{x_1[n] + x_1[-n]}{2}\left(1 + 2\cos(\frac{2\pi}{3}n)\right)e^{-j\Omega n}$$
$$x_2[n] = (x_1[n] + x_1[-n])\left(0.5 + \cos(\frac{2\pi}{3}n)\right)$$

(b) $Im\{X(\Omega)\} = X_o(\Omega) = \sum_{n=-\infty}^{\infty}\frac{x[n] - x[-n]}{2}e^{-j\Omega n}$

$$X_3(\Omega) = Im\{X_1(\Omega + \pi)\} + Im\{X_1(\Omega - \pi)\} = \sum_{n=-\infty}^{\infty}(x_1[n] - x_1[-n])(-1)^n e^{-j\Omega n}$$
$$x_3[n] = (-1)^n(x_1[n] - x_1[-n])$$

6.24

(ㄱ)

(a) $X(0) = \sum_{n=-\infty}^{\infty} x[n] = 12$

(b) $X(\pi) = \sum_{n=-\infty}^{\infty} (-1)^n x[n] = 0$

(c) $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\,d\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n}\,d\Omega\bigg|_{n=0} = x[0] = 1$

(d) $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\Omega)|^2\,d\Omega = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 28$

(ㄴ)

(a) $X(0) = \sum_{n=-\infty}^{\infty} x[n] = 12$

(b) $X(\pi) = \sum_{n=-\infty}^{\infty} (-1)^n x[n] = 0$

(c) $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\,d\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n}\,d\Omega\bigg|_{n=0} = x[0] = 3$

(d) $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\Omega)|^2\,d\Omega = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 28$

(ㄷ)

(a) $X(0) = \sum_{n=-\infty}^{\infty} x[n] = 0$

(b) $X(\pi) = \sum_{n=-\infty}^{\infty} (-1)^n x[n] = 4$

(c) $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\,d\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n}\,d\Omega\bigg|_{n=0} = x[0] = 1$

(d) $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\Omega)|^2\,d\Omega = \sum_{n=-\infty}^{\infty} |x[n]|^2 = 28$

6.25 $x'[n] = a^n u[n] * a^n u[n] = (n+1)a^n u[n]$이라 두면 시간 컨벌루션 성질에 의해서

$$X'(\Omega) = \frac{1}{1-ae^{-j\Omega}}\frac{1}{1-ae^{-j\Omega}} = \frac{1}{(1-ae^{-j\Omega})^2}$$

$ax'[n-1] = a \cdot na^{(n-1)}u[n-1] = na^n u[n-1] = na^n u[n] = x[n]$이므로 시간 이동 성질에 의해

$$X'(\Omega) = ae^{-j\Omega}X'(\Omega) = \frac{ae^{-j\Omega}}{(1-ae^{-j\Omega})^2}$$

6.26

(a)DTFT의 정의식으로부터

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$$= (\cdots + x[-k]\cos(\Omega k) + \cdots + x[0] + x[1]\cos(-\Omega) + \cdots + x[k]\cos(-\Omega k) + \cdots)$$

$$+ j(\cdots + x[-k]\sin(\Omega k) + \cdots + x[-1]\sin(\Omega) + x[1]\sin(-\Omega) + \cdots + x[k]\sin(-\Omega k) + \cdots)$$

$\sin(0) = 0$이므로 $\{x[i]\}$ 값에 상관없이 $Im\{X(0)\} = 0$이 된다.
따라서 $Re\{X(0)\} = 0$이 될 조건만 찾으면 된다. $x[n]$은 기함수 대칭을 만족하는 실수 기함수 신호이다.

(b) $$\int_{-\pi}^{\pi} X(\Omega)\,d\Omega = (2\pi)\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)\,e^{j\Omega n}\,d\Omega\right]_{n=0} = 2\pi x[0]$$

$$x[0] = 0$$

6.27

(a) $$H_1(\Omega) = \frac{1}{1-0.8e^{-j\Omega}}$$

$$|H_1(\Omega)| = \frac{1}{\sqrt{(1-0.8\cos\Omega)^2 + (0.8\sin\Omega)^2}}$$

진폭 스펙트럼은 저역 통과 특성을 나타낸다.

(1) $x[n] = (0.8)^n u[n]$

$$Y(\Omega) = H_1(\Omega)X(\Omega) = \frac{1}{1-0.8e^{-j\Omega}}\,\frac{1}{1-0.8e^{-j\Omega}}$$

$$y[n] = (n+1)(0.8)^n u[n]$$

(2) $x[n] = (-0.8)^n u[n]$

$$Y(\Omega) = H_1(\Omega)X(\Omega) = \frac{1}{1-0.8e^{-j\Omega}}\,\frac{1}{1+0.8e^{-j\Omega}} = \frac{0.5}{1-0.8e^{-j\Omega}} + \frac{0.5}{1+0.8e^{-j\Omega}}$$

$$y[n] = 0.5(0.8)^n u[n] + 0.5(-0.8)^n u[n]$$

(b) $$H_2(\Omega) = \frac{1}{1+0.8e^{-j\Omega}}$$

$$|H_2(\Omega)| = \frac{1}{\sqrt{(1+0.8\cos\Omega)^2 + (0.8\sin\Omega)^2}}$$

진폭 스펙트럼은 고역 통과 특성을 나타낸다.

(1) $x[n] = (0.8)^n u[n]$

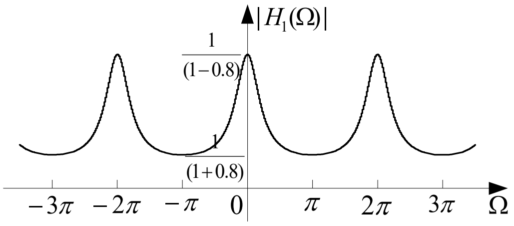
$$Y(\Omega) = H_2(\Omega)X(\Omega) = \frac{1}{1+0.8e^{-j\Omega}}\frac{1}{1-0.8e^{-j\Omega}} = \frac{0.5}{1-0.8e^{-j\Omega}} + \frac{0.5}{1+0.8e^{-j\Omega}}$$

$$y[n] = 0.5(0.8)^n u[n] + 0.5(-0.8)^n u[n]$$

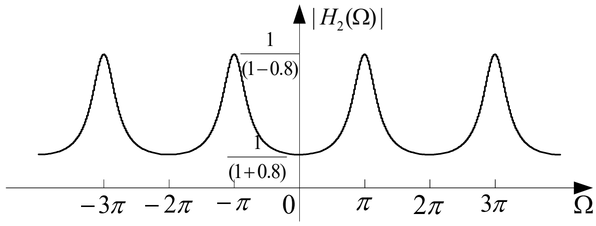
(2) $x[n] = (-0.8)^n u[n]$

$$Y(\Omega) = H_1(\Omega)X(\Omega) = \frac{1}{1+0.8e^{-j\Omega}}\frac{1}{1+0.8e^{-j\Omega}}$$

$$y[n] = (n+1)(-0.8)^n u[n]$$



(a)



(b)

$H_1(\Omega)$에 입력 (1)을 넣으면 출력 스펙트럼이 $|H_1(\Omega)|^2$과 같아져 커지지만, 입력 (2)를 넣으면 출력 스펙트럼이 $|H_1(\Omega)| \times |H_2(\Omega)|$가 되어 차단되는 특성을 보인다. 거꾸로 $H_2(\Omega)$에 입력 (2)를 넣으면 출력 스펙트럼이 $|H_2(\Omega)|^2$과 같아져 커지지만, 입력 (1)을 넣으면 출력 스펙트럼이 $|H_1(\Omega)| \times |H_2(\Omega)|$가 되어 차단되는 특성을 보인다. 즉 시스템이 저역 통과 특성이면 저주파 대역의 신호는 통과, 고주파 대역 통과 신호는 차단 효과를 가져오고, 시스템이 고역 통과 특성이면 저주파 대역의 신호는 차단, 고주파 대역 통과 신호는 통과 효과를 가져온다.

6.28 입력의 DTFT를 구하면

$$X_1(\Omega) = 2\pi\delta(\Omega) + \pi(\delta(\Omega+\frac{\pi}{6}) + \delta(\Omega-\frac{\pi}{6})) + \pi(\delta(\Omega+\frac{\pi}{3}) + \delta(\Omega-\frac{\pi}{3})) + j\pi(\delta(\Omega+\frac{5\pi}{6}) - \delta(\Omega-\frac{5\pi}{6}))$$

$$X_2(\Omega) = 1 + \pi(\delta(\Omega+\frac{\pi}{2}) + \delta(\Omega-\frac{\pi}{2}))$$

(a) (1) $x_1[n] = 1 + \cos(\frac{\pi}{6}n) + \cos(\frac{\pi}{3}n) + \sin(\frac{5\pi}{6}n)$

$$Y(\Omega) = H(\Omega)X(\Omega) = 2\pi\delta(\Omega) + \pi(\delta(\Omega+\frac{\pi}{6}) + \delta(\Omega-\frac{\pi}{6}))$$

$$y[n] = 1 + \cos(\frac{\pi}{6})$$

(2) $x[n] = \delta[n] + \cos(\frac{\pi}{2}n)$

$$Y(\Omega) = H(\Omega)X(\Omega) = H(\Omega)$$

$$y[n] = h[n] = \frac{1}{4}\mathrm{sinc}(\frac{n}{4})$$

(b) (1) $x_1[n] = 1 + \cos(\frac{\pi}{6}n) + \cos(\frac{\pi}{3}n) + \sin(\frac{5\pi}{6}n)$

$$Y(\Omega) = H(\Omega)X(\Omega) = \pi(\delta(\Omega + \frac{\pi}{3}) + \delta(\Omega - \frac{\pi}{3}))$$

$$y[n] = \cos(\frac{\pi}{3})$$

(2) $x[n] = \delta[n] + \cos(\frac{\pi}{2}n)$

$$Y(\Omega) = H(\Omega)X(\Omega) = H(\Omega) + \pi(\delta(\Omega + \frac{\pi}{2}) + \delta(\Omega - \frac{\pi}{2}))$$

$$y[n] = h[n] + \cos(\frac{\pi}{2}n) = \frac{1}{2}\mathrm{sinc}(\frac{n}{4})\cos(\frac{\pi}{2}n) + \cos(\frac{\pi}{2}n) = (1 + \frac{1}{2}\mathrm{sinc}(\frac{n}{4}))\cos(\frac{\pi}{2}n)$$

(c) (1) $x_1[n] = 1 + \cos(\frac{\pi}{6}n) + \cos(\frac{\pi}{3}n) + \sin(\frac{5\pi}{6}n)$

$$Y(\Omega) = H(\Omega)X(\Omega) = j\pi(\delta(\Omega + \frac{5\pi}{6}) - \delta(\Omega - \frac{5\pi}{6}))$$

$$y[n] = \sin(\frac{5\pi}{6})$$

(2) $x[n] = \delta[n] + \cos(\frac{\pi}{2}n)$

$$Y(\Omega) = H(\Omega)X(\Omega) = H(\Omega)$$

$$y[n] = h[n] = \frac{1}{2}\mathrm{sinc}(\frac{n}{4})\cos(\pi n) = (-1)^n\frac{1}{2}\mathrm{sinc}(\frac{n}{4})\cos(\pi n)$$

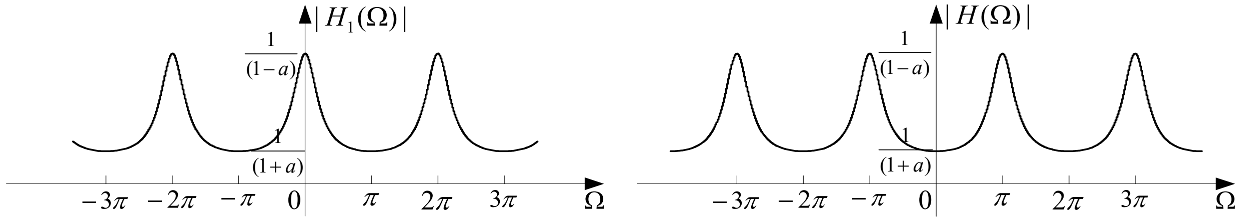
6.29

(a) $H(\Omega) = \dfrac{Y(\Omega)}{X(\Omega)} = H_1(\Omega - \pi)$

(b) $h[n] = (-1)^n h_1[n]$

(c) 고역 통과 필터

(d)



6.30

(a) $a = 1 - \epsilon,\ b = \epsilon\ (\because\ 1 - a \neq 0)$

(b) $a = -1 + \epsilon,\ b = -\epsilon\ (\because\ 1 + a \neq 0)$