Chapter 05 주파수 영역 해석의 기초

[Quick Review]

(1) 변환

(2) ○

(3) ×

(4) ×

(5) ○

(6) ×

(7) ×

(8) 빠르다

(9) 없다

(10) 직류

(11) ×

(12) 3θ

(13) ×

(14) 연속

(15) ×

(16) 에너지

(17) ○

(18) 같다.

(19) ○

(20) 대역 이동

[기초 문제]

5.1 ㉮, ㉯

5.2 ㉰

5.3 ㉰

5.4 ㉱

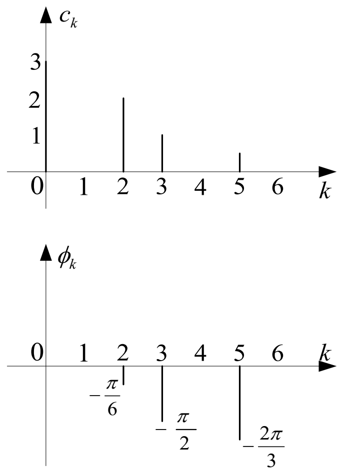
5.5 ㉯, ㉱

5.6

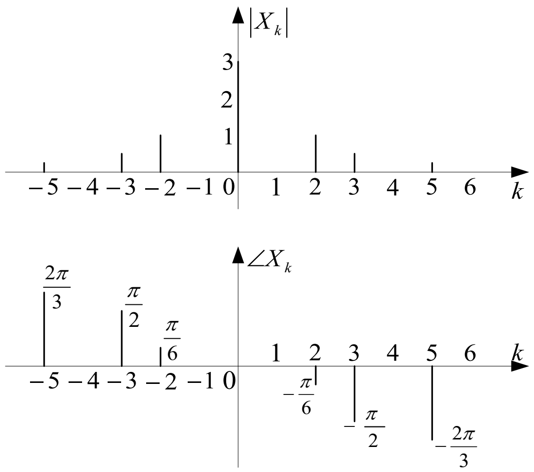
(a) $x(t) = 3 + 2\cos(2t - \frac{\pi}{6}) + \cos(3t - \frac{\pi}{2}) + \frac{1}{2}\cos\left(5t - \frac{2\pi}{3}\right)$

(b) $X_0 = c_0$, $|X_k| = c_k/2$, $\angle X_k = \phi_k$

(c) $x(t) = \frac{1}{4}e^{j\frac{2\pi}{3}}e^{-j5t} + \frac{1}{2}e^{j\frac{\pi}{2}}e^{-j3t} + e^{j\frac{\pi}{6}}e^{-j2t} + 3 + e^{-j\frac{\pi}{6}}e^{j2t} + \frac{1}{2}e^{-j\frac{\pi}{2}}e^{j3t} + \frac{1}{4}e^{-j\frac{2\pi}{3}}e^{j5t}$



삼각함수형 Fourier 스펙트럼



지수함수형 Fourier 스펙트럼

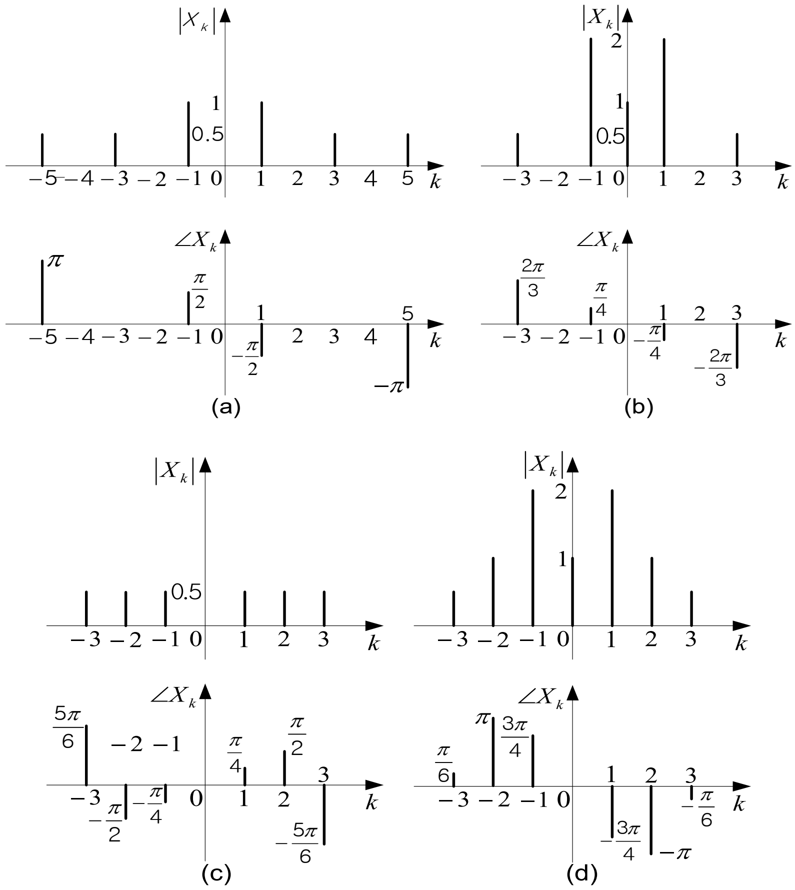
5.7

(a) $x(t)=\frac{1}{2}e^{j\pi}e^{-j5\pi t}+\frac{1}{2}e^{-j3\pi t}+e^{j\frac{\pi}{2}}e^{-j\pi t}+e^{-j\frac{\pi}{2}}e^{j\pi t}+\frac{1}{2}e^{j3\pi t}+\frac{1}{2}e^{-j\pi}e^{j5\pi t}$

(b) $x(t)=\frac{1}{2}e^{j\frac{2\pi}{3}}e^{-j3\pi t}+2e^{j\frac{\pi}{4}}e^{-j\pi t}+1+2e^{-j\frac{\pi}{4}}e^{j\pi t}+\frac{1}{2}e^{-j\frac{2\pi}{3}}e^{j3\pi t}$

(c) $x(t)=\frac{1}{2}e^{j\frac{5\pi}{6}}e^{-j6\pi t}+\frac{1}{2}e^{-j\frac{\pi}{2}}e^{-j4\pi t}+\frac{1}{2}e^{-j\frac{\pi}{4}}e^{-j2\pi t}+\frac{1}{2}e^{j\frac{\pi}{4}}e^{j2\pi t}+\frac{1}{2}e^{j\frac{\pi}{2}}e^{j4\pi t}+\frac{1}{2}e^{-j\frac{5\pi}{6}}e^{j6\pi t}$

(d) $x(t)=\frac{1}{2}e^{j\frac{\pi}{6}}e^{-j3t}+e^{j\pi}e^{-j2t}+2e^{j\frac{3\pi}{4}}e^{-jt}+1+2e^{-j\frac{3\pi}{4}}e^{jt}+e^{-j\pi}e^{j2t}+\frac{1}{2}e^{-j\frac{\pi}{6}}e^{j3t}$



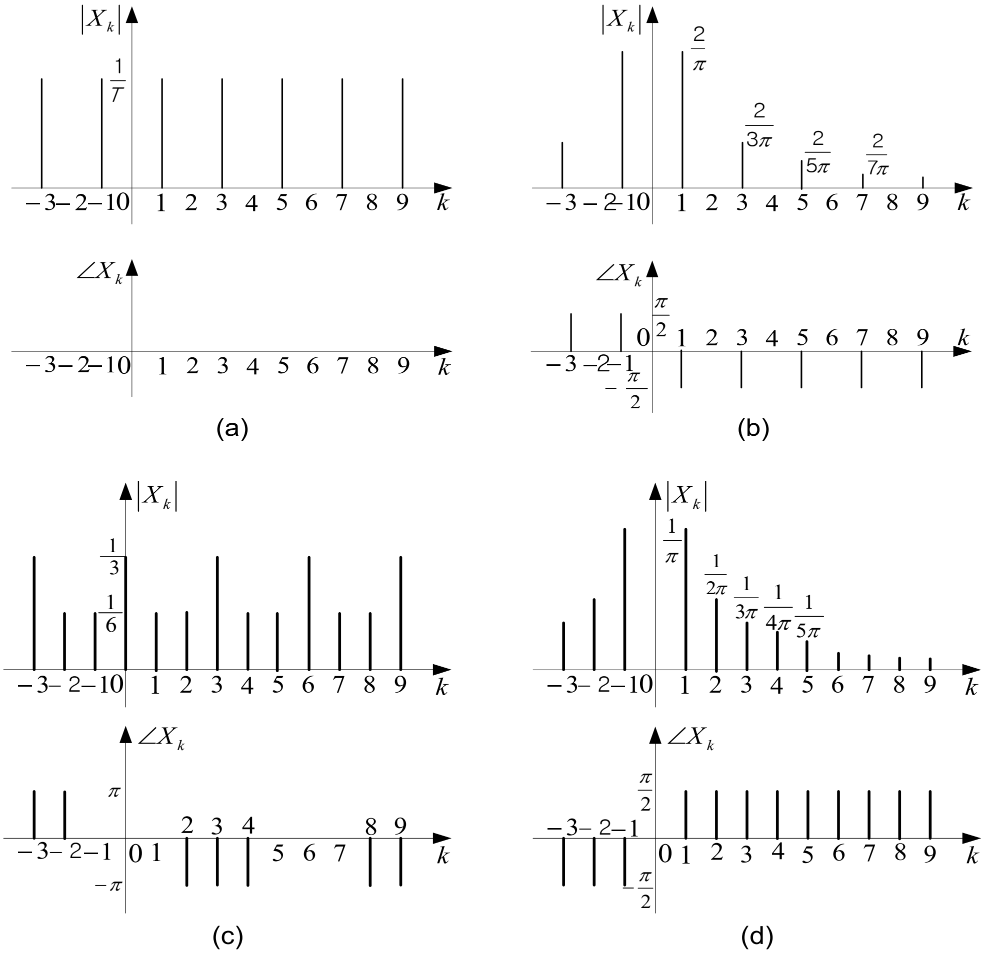
5.8

(a) $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} \frac{1}{2T}(1-e^{-jk\pi})e^{jk\omega_0 t}$

(b) $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t} = \sum_{k=\text{홀수}} -j\frac{2}{k\pi}e^{jk\omega_0 t}$

(c) $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} \frac{1}{3}\cos(\frac{\pi}{3}k)e^{jk\frac{\pi}{3}t}$

(d) $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t} = \sum_{k\neq 0,\, k=-\infty}^{\infty} j\frac{1}{k\pi}e^{jk\pi t}$



5.9

(a) $x(t) = X_0 + \sum_k 2|X_k|\cos(k\omega_0 t + \angle X_k) = 3 + 4\cos\left(\pi t - \frac{\pi}{3}\right) + 2\cos\left(3\pi t - \frac{\pi}{2}\right)$

(b) $x(t) = X_0 + \sum_k 2|X_k|\cos(k\omega_0 t + \angle X_k) = 2 + 2\sqrt{2}\cos\left(2\pi t + \frac{\pi}{4}\right) + 2\cos\left(4\pi t + \frac{\pi}{2}\right)$

5.10

(a) $x(t) = X_0 + \sum_k 2|X_k|\cos(k\omega_0 t + \angle X_k) = 1 + 4\cos\left(\pi t - \frac{\pi}{6}\right) - 2\cos(2\pi t) + 4\cos\left(3\pi t - \frac{3\pi}{4}\right)$

(b) $x(t) = X_0 + \sum_k 2|X_k|\cos(k\omega_0 t + \angle X_k) = 2 + 4\cos\left(2\pi t - \frac{\pi}{4}\right) + 2\cos\left(4\pi t + \frac{\pi}{3}\right) + 2\sin(6\pi t)$

(c) $x(t) = X_0 + \sum_k 2|X_k|\cos(k\omega_0 t + \angle X_k) = 1 + 2\sin(\pi t) - 2\cos(2\pi t) - 2\sin(3\pi t) + 2\cos(4\pi t)$

(d) $x(t) = X_0 + \sum_k 2|X_k|\cos(k\omega_0 t + \angle X_k) = 16 + 12\cos(3t - \frac{\pi}{4}) + 8\sin(6t) + 4\cos(9t - \frac{\pi}{4})$

5.11 $P_x = |X_{-1}|^2 + |X_1|^2 = \frac{A^2}{4} + \frac{A^2}{4} = \frac{A^2}{2}$

5.12

(a) $P = \sum_{k=0}^{\infty} \bar{c}_k^2 = 1^2 + (\frac{2}{\sqrt{2}})^2 + (\frac{4}{\sqrt{2}})^2 = 11$

(b) $P = \sum_{k=0}^{\infty} \bar{c}_k^2 = 2^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{2}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 = 7$

(c) $P = \sum_{k=0}^{\infty} \bar{c}_k^2 = 2^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{3}{\sqrt{2}})^2 + (\frac{4}{\sqrt{2}})^2 = \frac{35}{2} = 17\frac{1}{2}$

(d) $P = \sum_{k=0}^{\infty} \bar{c}_k^2 = 1^2 + (\frac{2}{\sqrt{2}})^2 + (\frac{2\sqrt{3}}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 = 10$

5.13

(a) $X(\omega) = \int_0^T e^{-at}e^{-j\omega t}dt = -\frac{e^{-(a+j\omega)T} - 1}{a + j\omega}$

(b) $X(\omega) = \int_{-1}^{1} \sin(2\pi t)e^{-j\omega t}dt = \frac{1}{2}(e^{j\omega} - e^{-j\omega})(\frac{1}{\omega + 2\pi} - \frac{1}{\omega - 2\pi}) = j\sin(\omega)\frac{4\pi}{4\pi^2 - \omega^2}$

(c) $X(\omega) = 4\int_0^1 e^{-j\omega t}dt + 2\int_1^2 e^{-j\omega t}dt = \frac{4 - 2(e^{-j\omega} + e^{-j2\omega})}{j\omega}$

(d) $X(\omega) = \int_{-\tau}^{0}\left(-\frac{1}{\tau}t\right)e^{-j\omega t}dt + \int_0^{\tau}\frac{1}{\tau}t\,e^{-j\omega t}dt = \frac{1 - j\tau\omega}{\tau\omega^2}(e^{j\omega\tau} + e^{-j\omega\tau})$

5.14

(a) $x(t) = \frac{1}{2\pi}\int_{-3}^{-1} 2\pi e^{j\omega t}d\omega + \frac{1}{2\pi}\int_1^3 2\pi e^{j\omega t}d\omega = \frac{2}{t}(\sin(3t) - \sin(t))$

(b) $x(t) = \frac{1}{2\pi}\int_{-2}^{-1} e^{j\omega t}d\omega + \frac{1}{2\pi}\int_{-1}^{1} 2e^{j\omega t}d\omega + \frac{1}{2\pi}\int_1^2 e^{j\omega t}d\omega = \frac{1}{\pi t}(\sin(2t) + \sin(t))$

(c) $x(t) = \left[\mathrm{sinc}\left(\frac{1}{2}(t+1)\right) + \mathrm{sinc}\left(\frac{1}{2}(t-1)\right)\right]$

5.15

(a) $X(\omega) = U(\omega) - U(\omega)e^{-j4\omega} = \left(\pi\delta(\omega) + \frac{1}{j\omega}\right)(1 - e^{-j4\omega}) = \frac{2e^{-j2\omega}}{\omega}\sin(2\omega)$

(b) $X(\omega) = V(\omega) - e^{-8}V(\omega)e^{-j4\omega} = \frac{1}{j\omega + 2}(1 - e^{-4(j\omega+2)})$

(c) $X(\omega) = j\frac{dV(\omega)}{d\omega} = \frac{1}{\omega^2}(e^{-j4\omega} - 1) + j\frac{4}{\omega}e^{-j4\omega} = \frac{e^{-j4\omega}(1 + j4\omega) - 1}{\omega^2}$

5.16

(a) $X(\omega) = \frac{1}{2 + j\omega} + \frac{1}{2 - j\omega} = \frac{4}{4 + \omega^2}$

(b) $X(\omega) = (j\omega)V(\omega) = (j\omega)\frac{4}{4 + \omega^2} = \frac{j4\omega}{4 + \omega^2}$

(c) $X(\omega) = (j\omega)V(\omega) = \frac{j\omega}{(2 + j\omega)^2}$

(d) $X(\omega) = \frac{1}{2\pi}\mathrm{rect}(\omega/2\pi) * \mathrm{rect}(\omega/2\pi) = \mathrm{tri}(\omega/2\pi)$

(e) $X(\omega) = \mathrm{rect}(\omega/2\pi) \cdot \mathrm{rect}(\omega/2\pi) = \mathrm{rect}(\omega/2\pi)$

(f) $X(\omega) = \frac{\omega_0}{j2\pi\omega + \omega_0^2 - \omega^2 + \pi^2}$

5.17

(a) $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \frac{1}{j\omega}(2 - e^{j\omega} - e^{-j\omega}) = j\frac{2}{\omega}(\cos\omega - 1)$

(b) $X(\omega) = -\frac{1}{j\omega}(e^{j\omega} + e^{-j\omega} - 2) = \frac{2}{j\omega}(1 - \cos\omega)$

(c) $X(\omega) = \frac{1}{j\omega}X'(\omega) = \frac{2}{j\omega}(1 - \cos\omega)$

(d) $X(\omega) = \mathrm{sinc}(\frac{\omega}{2\pi})(e^{-j\frac{1}{2}\omega} - e^{j\frac{1}{2}\omega}) = \frac{2}{j\omega}(\cos(0) - \cos(\omega)) = \frac{2}{j\omega}(1 - \cos(\omega))$

5.18

(a) $x(t) = e^{-at}u(t) * e^{-at}u(t) = \; = te^{-at}u(t)$

(b) $x(t) = tv(t) = te^{-at}u(t)$

(c) $x(t) = \frac{1}{(N-1)!(-j)^{N-1}}(-j)^{N-1}t^{N-1}v(t) = \frac{1}{(N-1)!}t^{N-1}e^{-at}u(t)$

5.19

(a) $x(t) = [\delta(t+2)+\delta(t-2)] * \frac{1}{2}\mathrm{rect}(\frac{t}{2}) = \frac{1}{2}[\mathrm{rect}(\frac{t+2}{2})+\mathrm{rect}(\frac{t-2}{2})]$

(b) $x(t) = \frac{1}{2\pi}\frac{2a}{a^2+t^2} = \frac{a}{\pi(a^2+t^2)}$

(c) $x(t) = \frac{1}{2}\mathrm{rect}(\frac{t}{2})(e^{j2\pi t}+e^{-j2\pi t}) = \mathrm{rect}(\frac{t}{2})\cos(2\pi t)$

(d) $x(t) = j\frac{1}{2\pi}\frac{2}{jt} = \frac{1}{\pi t}$

(e) $x(t) = (\delta(t+1)+\delta(t-1)) * \mathrm{sinc}(t) = \mathrm{sinc}(t+1)+\mathrm{sinc}(t-1) = \frac{\sin(\pi(t+1))}{\pi(t+1)} + \frac{\sin(\pi(t-1))}{\pi(t-1)}$

(f) $x(t) = \frac{dv(t)}{dt} = \frac{1}{\pi t}\cos(t) - \frac{1}{\pi t^2}\sin(t)$

5.20

(a) $E = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\omega = 1$

(b) $E = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\pi^2\mathrm{tri}^2(\frac{\omega}{2})\,d\omega = \frac{8\pi}{3}$

(c) $E = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega = 2\pi\int_{0}^{\infty} e^{-2a\omega} d\omega = \frac{2\pi}{a}$

(d) $E = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4\times 2^2}{(2^2+\omega^2)^2}d\omega = \frac{1}{2\pi}(\frac{\pi}{2}+\frac{\pi}{2}) = \frac{1}{2}$

※ 위의 적분은 $\omega = 2\tan\theta$로 치환하여 계산하면 된다.
※ 이 문제는 특수 치환 적분 때문에 계산이 까다로워, 시간 영역 적분으로 에너지를 계산하는 것이 더 쉽다.

$$E = \int_{-\infty}^{\infty}|x(t)|^2 dt = \int_{-\infty}^{\infty} e^{-4|t|}dt = \int_{-\infty}^{0} e^{4t}dt + \int_{0}^{\infty} e^{-4t}dt = \frac{1}{4}e^{4t}\Big|_{-\infty}^{0} - \frac{1}{4}e^{-4t}\Big|_{0}^{\infty} = \frac{1}{4}+\frac{1}{4} = \frac{1}{2}$$

[응용 문제]

5.21

(a) $\omega_0 = 2$ [rad/sec]

삼각함수 형식 푸리에 급수

$$x(t) = \sin\left(2t + \frac{\pi}{4}\right) = \cos\left(2t - \frac{\pi}{4}\right) = \sum_{k=0}^{\infty} c_k \cos k\omega_o t$$

$$\therefore\ c_1 = 1,\quad \theta_1 = -\frac{\pi}{4}$$

지수 함수 형식 푸리에 급수

$$x(t) = \sin(2t + \frac{\pi}{4}) = \frac{1}{2}e^{-j\frac{\pi}{4}}e^{j2t} + \frac{1}{2}e^{j\pi}e^{-j\frac{3\pi}{4}}e^{-j2t} = \sum_{k=-\infty}^{\infty} X_k e^{jk2t}$$

$$\therefore\ X_{-1} = \frac{1}{2}e^{j\frac{\pi}{4}},\quad X_1 = \frac{1}{2}e^{-j\frac{\pi}{4}}$$

(b) $\omega_0 = 4$ [rad/sec]

삼각함수 형식 푸리에 급수

전파 정류 파형은 우대칭이므로 사인파 성분은 존재하지 않고 DC 성분과 코사인파 성분만 존재한다.

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_o t = \sum_{k=0}^{\infty} c_k \cos k\omega_o t$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{2}{\pi}$$

$$a_k = \frac{2}{T}\int_0^T x(t)\cos k\omega_0 t\,dt = \frac{1}{(2k-1)\pi}(-1-1) - \frac{1}{(2k+1)\pi}(-1-1) = \frac{4}{(1-4k^2)\pi}$$

지수 함수 형식 푸리에 급수

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j4kt}$$

$$X_0 = a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{2}{\pi}$$

$$X_k = \frac{1}{T}\int_0^T x(t)e^{-j4kt}\,dt = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{(e^{j2t} - e^{-j2t})}{2j} e^{-j4kt}\,dt = \frac{2}{(1-4k^2)\pi}$$

(c) $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$ [rad/sec]

삼각함수 형식 푸리에 급수

$$x(t) = \cos(2t - \frac{\pi}{2}) + \cos 4t = \sum_{k=0}^{\infty} c_k \cos(k\omega_0 t + \theta_k)$$

$$\therefore\ c_1 = 1,\quad \theta_1 = -\frac{\pi}{2} \quad \& \quad c_2 = 1,\quad \theta_2 = 0$$

지수 함수 형식 푸리에 급수

$$x(t)=\frac{1}{2}e^{-j4t}-\frac{1}{2j}e^{-j2t}+\frac{1}{2j}e^{j2t}+\frac{1}{2}e^{j4t}=\sum_{k=-\infty}^{\infty}X_k e^{j2kt}$$

$$\therefore\ X_{-2}=\frac{1}{2},\quad X_{-1}=-\frac{1}{2j}=\frac{1}{2}e^{j\frac{\pi}{2}},\quad X_1=\frac{1}{2j}=\frac{1}{2}e^{-j\frac{\pi}{2}},\quad X_2=\frac{1}{2}$$

(d) $\omega_0=\frac{2\pi}{T}=\frac{2\pi}{\pi}=2$ [rad/sec]

삼각함수 형식 푸리에 급수

$$x(t)=-\frac{1}{2}\cos(2t-\frac{\pi}{2})+\frac{1}{2}\cos(8t-\frac{\pi}{2})=\frac{1}{2}\cos(2t+\frac{\pi}{2})+\frac{1}{2}\cos(8t-\frac{\pi}{2})=\sum_{k=0}^{\infty}c_k\cos(k\omega_0 t+\theta_k)$$

$$\therefore\ c_1=\frac{1}{2},\quad \theta_1=\frac{\pi}{2}\qquad \&\qquad c_4=\frac{1}{2},\quad \theta_4=-\frac{\pi}{2}$$

지수 함수 형식 푸리에 급수

$$x(t)=\frac{1}{4}e^{j\frac{\pi}{2}}e^{-j8t}+\frac{1}{4}e^{-j\frac{\pi}{2}}e^{-j2t}+\frac{1}{4}e^{j\frac{\pi}{2}}e^{j2t}+\frac{1}{4}e^{-j\frac{\pi}{2}}e^{j8t}=\sum_{k=-\infty}^{\infty}X_k e^{j2kt}$$

$$\therefore\ X_{-4}=\frac{1}{4}e^{j\frac{\pi}{2}},\quad X_{-1}=\frac{1}{4}e^{-j\frac{\pi}{2}},\quad X_1=\frac{1}{4}e^{j\frac{\pi}{2}},\quad X_4=\frac{1}{4}e^{-j\frac{\pi}{2}}$$

5.22

(a) $$X_k=\begin{cases}-j\frac{1}{4}, & k=1\\ \frac{1}{(1-k^2)\pi}, & k=even\\ 0, & k=odd,\ k\neq 1\end{cases}\qquad \&\qquad X_{-k}=X_k^*$$

(b) $$Y_k=\frac{1}{T}\int_0^T y(t)e^{-j4\pi kt}dt=\frac{2}{(1-4k^2)\pi}e^{-jk\pi}\cos k\pi=\frac{2}{(1-4k^2)\pi}$$

(c) $$y(t)=x(t)+x(t-\frac{1}{2})=\sum_{k=-\infty}^{\infty}(1+(-1)^k)X_k e^{j2\pi kt}=\sum_{k'=-\infty}^{\infty}\frac{2}{(1-4k'^2)\pi}e^{j4\pi k't}$$

5.23

(a) $s(t)=x(t)-1$

따라서 $s(t)$의 스펙트럼은 $x(t)$의 스펙트럼에서 진폭 스펙트럼의 DC 성분이 달라진다.

$$S_0=X_0-1=\mathrm{sinc}(0)-1=1-1=0$$

$$S_k=X_k=\mathrm{sinc}\left(\frac{k}{2}\right),\quad k\neq 0$$

(b) $v(t)=x(t-1)-1$

따라서 $v(t)$의 스펙트럼은 $x(t)$의 스펙트럼에서 진폭 스펙트럼의 DC 성분 및 위상 스펙트럼이 달라진다.

$$V_0=X_0-1=\mathrm{sinc}(0)-1=1-1=0$$

$$V_k=e^{-j\frac{\pi}{2}k}X_k=e^{-j\frac{\pi}{2}k}\mathrm{sinc}\left(\frac{k}{2}\right)=-j\frac{1}{k\pi}(1-e^{-j\pi k})=\begin{cases}-j\frac{2}{k\pi}, & k=\text{홀수}\\ 0, & k=\text{짝수}\end{cases}$$

(c) $w(t)=x(t+1)-2$

따라서 $w(t)$의 스펙트럼은 $x(t)$의 스펙트럼에서 진폭 스펙트럼의 DC 성분 및 위상 스펙트럼이 달라진다.

$$W_0=X_0-2=\mathrm{sinc}(0)-1=1-2=-1$$

$$W_k=e^{j\frac{\pi}{2}k}X_k=e^{j\frac{\pi}{2}k}\mathrm{sinc}\left(\frac{k}{2}\right)=-j\frac{1}{k\pi}(e^{j\pi k}-1)=\begin{cases}j\frac{2}{k\pi}, & k=\text{홀수}\\ 0, & k=\text{짝수}\end{cases}$$

(d) $y(t)=2x(t-1)-2$

따라서 $y(t)$의 스펙트럼은 $x(t)$의 스펙트럼에서 진폭 스펙트럼의 DC 성분 및 위상 스펙트럼이 달라진다.

$$Y_0=2X_0-2=2\mathrm{sinc}(0)-2=2-2=0$$

$$Y_k=2e^{-j\frac{\pi}{2}k}X_k=2e^{-j\frac{\pi}{2}k}\mathrm{sinc}\left(\frac{k}{2}\right)=-j\frac{2}{k\pi}(1-e^{-j\pi k})=\begin{cases}-j\frac{4}{k\pi}, & k=\text{홀수}\\ 0, & k=\text{짝수}\end{cases}$$

(e) $z(t)=x(t-2)-1$

$$Z_0=X_0-1=\mathrm{sinc}(0)-1=1-1=0$$

$$Z_k=e^{-j\pi k}X_k=e^{-j\pi k}\mathrm{sinc}\left(\frac{k}{2}\right)=\begin{cases}\mathrm{sinc}\left(\frac{k}{2}\right), & k=\text{짝수}\\ -\mathrm{sinc}\left(\frac{k}{2}\right), & k=\text{홀수}\end{cases}$$

5.24

(a) $$X_k=\frac{1}{T}\int_{-T/2}^{T/2}x(t)e^{-jk\omega_0 t}dt=j\frac{5}{k\pi}(e^{-jk\pi}-1)=\begin{cases}-j\frac{10}{k\pi}, & k=\text{홀수}\\ 0, & k=\text{짝수}\end{cases}$$

$$X_0=\frac{1}{T}\int_T x(t)dt=0$$

(b) $$P=|X_0|^2+\sum_{k=1}^{\infty}2|X_k|^2=\sum_{k>0,\,k=\text{홀수}}^{\infty}2\left(\frac{10}{k\pi}\right)^2=\frac{200}{\pi^2}\sum_{k>0,\,k=\text{홀수}}^{\infty}\frac{1}{k^2}=\frac{200}{\pi^2}\frac{\pi^2}{8}=25$$

5.25

(a) $$x(t)=\frac{1}{2\pi}\int_{-1}^{1}e^{-j\frac{\pi}{2}\omega}e^{j\omega t}d\omega=\frac{\sin(t-\frac{\pi}{2})}{\pi(t-\frac{\pi}{2})}=\frac{1}{\pi}\mathrm{sinc}\left(\frac{t}{\pi}-\frac{1}{2}\right)$$

(b) $$x(t)=\frac{1}{2\pi}\left[\int_{-1}^{1}je^{j\omega t}d\omega+\int_{0}^{1}-je^{j\omega t}d\omega\right]=\frac{1-\cos t}{\pi t}$$

5.26

(a) $$X_1(\omega)=X(-\omega)=\frac{1}{\omega^2}\left[e^{-j\omega}+j\omega e^{-j\omega}-1\right]$$

(b) $$X_2(\omega)=X(\omega)e^{-j\omega}+X(-\omega)e^{j\omega}=\frac{1}{\omega^2}[2-2\cos\omega]=\frac{1}{2}\mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$$

(c) $$X_3(\omega)=X(\omega)+X_1(\omega)+X_2(\omega)=\frac{1}{\omega^2}[-j\omega(e^{j\omega}-e^{-j\omega})]=\frac{2}{\omega}\sin(\omega)=2\mathrm{sinc}\left(\frac{\omega}{\pi}\right)$$

5.27

(a) $\dfrac{1}{2+j\omega}$

(b) $\dfrac{e^{-j2\omega}}{3+j\omega}$

(c) $\dfrac{e^{-j2\omega}}{1-\omega^2+j2\omega}$

(d) $\dfrac{1}{2(\omega+2\pi)-j2}-\dfrac{1}{2(\omega-2\pi)-j2}$

5.28

(a) $X(\omega)=2\cos(t_0\omega)$

(b) $X(\omega)=2\pi e^{\omega}u(-\omega)$

(c) $X(\omega)=2\pi e^{-a|\omega|}$

5.29

(a) $x(t+T)+x(t-T) \Leftrightarrow X(\omega)e^{jT\omega}+X(\omega)e^{-jT\omega}=X(\omega)(e^{jT\omega}+e^{-jT\omega})=2X(\omega)\cos(T\omega)$

(b) (ㄱ) $X(\omega) = 2[2\mathrm{sinc}(\frac{\omega}{\pi})]\cos(3\omega)=4\mathrm{sinc}(\frac{\omega}{\pi})\cos(3\omega)$

(ㄴ) $X(\omega) = 2\mathrm{sinc}^2(\frac{\omega}{2\pi})\cos(3\omega)$

(c) (ㄱ) $x(t) = 2[\frac{2}{\pi}\mathrm{sinc}(\frac{t}{\pi})]\cos(3t)=\frac{4}{\pi}\mathrm{sinc}(\frac{t}{\pi})\cos(3t)$

(ㄴ) $x(t) = 2\frac{1}{2\pi}\mathrm{sinc}^2(\frac{t}{2\pi})\cos(3t) = \frac{1}{\pi}\mathrm{sinc}^2(\frac{t}{2\pi})\cos(3t)$

5.30 $x_1(t) = 2x(t)(\cos\omega_0 t)(\cos\omega_0 t) = 2x(t)\cos^2\omega_0 t = x(t)(1+\cos 2\omega_0 t)=x(t)+x(t)\cos 2\omega_0 t$

$$X_1(\omega) = X(\omega)+X(\omega)*(\pi[\delta(\omega+2\omega_0)+\delta(\omega-2\omega_0)] = X(\omega)+\pi X(\omega+2\omega_0)+\pi X(\omega+2\omega_0)$$

대역폭이 W인 저역 통과 필터에 $x_1(t)$를 통과시키면 $X_1(\omega)$에서 $X(\omega\pm 2\omega_0)$ 성분은 필터의 통과 대역 바깥에 위치하므로 제거되고, $X(\omega)$만 남게 된다. 따라서 $x(t)$를 필터의 출력으로 얻게 되어 신호 $x(t)$를 복조할 수 있다.