

<4장 연습문제 정답>

연습문제 4.1

1. (a) $y' = 6 \cos 6x$ (b) $y' = \frac{1}{5} \cos \left(\frac{1}{5}x + 3 \right)$
 (c) $y' = -\pi^3 \sin \pi^3 x$ (d) $y' = -\sqrt{11} \sin \sqrt{11} (x - 2)$
 (e) $y' = \frac{5}{3} \sec^2 \frac{5}{3} x$ (f) $y' = \frac{1}{\sqrt{2}} \sec^2 \frac{1}{\sqrt{2}} (x + \sqrt{3})$
 (g) $y' = -\frac{1}{\pi} \csc \frac{1}{\pi} x \cot \frac{1}{\pi} x$ (h) $y' = \csc (3 - x) \cot (3 - x)$
 (i) $y' = \sqrt{\frac{3}{2}} \sec \sqrt{\frac{3}{2}} x \tan \sqrt{\frac{3}{2}} x$ (j) $y' = -\frac{1}{\pi^2} \sec \left(\pi - \frac{1}{\pi^2} x \right) \tan \left(\pi - \frac{1}{\pi^2} x \right)$
 (k) $y' = -\sqrt{\pi} \csc^2 \sqrt{\pi} x$ (l) $y' = \pi^5 \csc^2 (3 - \pi^5 x)$

3. (a) $y' = 7 \sin^6 x \cos x$ (b) $y' = 7x^6 \cos x^7$
 (c) $y' = -6 \cos^5 x \sin x$ (d) $y' = -6x^5 \sin x^6$
 (e) $y' = 9 \tan^8 x \sec^2 x$ (f) $y' = 9x^8 \sec^2 x^9$
 (g) $y' = 8 \sec^8 x \tan x$ (h) $y' = 8x^7 \sec x^8 \tan x^8$

5. (a) $y' = 6 \sin (3x + 1) \cos (3x + 1)$
 (b) $y' = -30 \cos^4 (6x - 5) \sin (6x - 5)$
 (c) $y' = \frac{7}{2} \tan^{\frac{5}{2}} x \sec^2 x$
 (d) $y' = \frac{2}{3} \sin^{-\frac{1}{3}} x \cos x$

$$(e) y' = -24 \sin^7(\cos 3x) \cos(\cos 3x) \sin 3x$$

$$(f) y' = 20 \tan^4(\sin^4 x) \sec^2(\sin^4 x) \sin^3 x \cos x$$

$$(g) y' = -4 \cos(\tan^2 x + 1) \sin(\tan^2 x + 1) \tan x \sec^2 x$$

$$(h) y' = -6 \tan(\cos^3 x - 1) \sec^2(\cos^3 x - 1) \cos^2 x \sin x$$

$$(i) y' = 14 \sec^2(\sin^7 x + 3) \tan(\sin^7 x + 3) \sin^6 x \cos x$$

$$(j) y' = -18 \csc^3(\sin^6 x - 9) \cot(\sin^6 x - 9) \sin^5 x \cos x$$

연습문제 4.2

$$1. (a) y' = \frac{1}{x} \frac{1}{\ln 7}$$

$$(b) y' = \frac{1}{x} \frac{1}{\ln 1.6}$$

$$(c) y' = \frac{1}{x} \frac{1}{\ln 10}$$

$$(d) y' = \frac{1}{x} \frac{1}{\ln \frac{1}{\sqrt{2}}} = -\frac{1}{x} \frac{1}{\ln \sqrt{2}}$$

$$3. (a) y' = \frac{12x^2}{(4x^3 + 1) \ln 2}$$

$$(b) y' = \frac{2x + \cos x}{(x^2 + \sin x) \ln 0.3}$$

$$(c) y' = \frac{-\sin x + 6x^2}{(\cos x + 2x^3) \ln 5}$$

$$(d) y' = \frac{-1 - 3x^2}{(1 - x - x^3) \ln \frac{2}{7}}$$

$$5. (a) y' = 2.3e^{2.3x}$$

$$(b) y' = (2x + \sqrt{3})e^{x^2 + \sqrt{3}x}$$

$$(c) y' = \cos x e^{\sin x}$$

$$(d) y' = -\frac{1}{x^2} e^{\frac{1}{x}}$$

7. (a) $y' = (3x+5)^{10} (x^2-1)^{11} \left(\frac{30}{3x+5} + \frac{22x}{x^2-1} \right)$

(b) $y' = \frac{(3x^2-1)^6}{(5x+4)^{11}} \left(\frac{36x}{3x^2-1} - \frac{55}{5x+4} \right)$

(c) $y' = \frac{1}{2} \sqrt{\frac{x^2-1}{x^3+1}} \left(\frac{2x}{x^2-1} - \frac{3x^2}{x^3+1} \right)$

(d) $y' = \frac{1}{3} \sqrt[3]{\frac{2x^2-x-1}{x^5+4x}} \left(\frac{4x-1}{2x^2-x-1} - \frac{5x^4+4}{x^5+4x} \right)$

(e) $y' = x^4 \sqrt{\frac{x-2}{x+1}} \frac{4}{x} + \frac{1}{2} \left(\frac{1}{x-2} - \frac{1}{x+1} \right)$

(f) $y' = \frac{\sqrt{x^2-4}}{x^3\sqrt{x^2+4}} \left(\frac{x}{x^2-4} - \frac{3}{x} - \frac{x}{x^2+4} \right)$

9. (a) $y' = \sqrt{5} x^{\sqrt{5}-1}$

(b) $y' = (\sqrt{7} - \sqrt{5}) x^{\sqrt{7}-\sqrt{5}-1}$

(c) $y' = -\pi x^{-\pi-1}$

(d) $y' = \left(\pi^2 + \frac{1}{e} \right) x^{\pi^2 + \frac{1}{e} - 1}$

(e) $y' = (e^2 + 3) x^{e^2+2}$

(f) $y' = \ln 9 x^{\ln 9 - 1}$

연습문제 4.3

1. (a) 0

(b) 1

(c) 0

(d) $\frac{3}{4}$

(e) $\frac{5}{3}$

(f) $\frac{99}{101}$

3. (a) $\lim_{x \rightarrow \infty} \sinh x$

(b) $\lim_{x \rightarrow \infty} \cosh x$

(c) $\lim_{x \rightarrow \infty} \tanh x$

(d) $\lim_{x \rightarrow -\infty} \sinh x$

(e) $\lim_{x \rightarrow -\infty} \cosh x$

(f) $\lim_{x \rightarrow -\infty} \tanh x$

5. (a) $y' = \cosh(\cosh x) \sinh x$

(b) $y' = \sinh(\sinh x) \cosh x$

(c) $y' = \operatorname{sech}^2(\sinh x) \cosh x$

(d) $y' = \operatorname{sech}^2(\cosh x) \sinh x$

(e) $y' = 3 \sinh^2(x^3 + 2x) \cosh(x^3 + 2x) (3x^2 + 2)$

(f) $y' = -8x \cosh^3(3 - x^2) \sinh(3 - x^2)$

(g) $y' = 4(e^x + 2x) \tanh^3(e^x + x^2) \operatorname{sech}^2(e^x + x^2)$

(h) $y' = -\frac{3}{x^2} \tanh^2 \frac{1}{x} \operatorname{sech}^2 \frac{1}{x}$

7. (a) $(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$

(b) $(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$

(c) $(\coth x)' = -\operatorname{csch}^2 x$

연습문제 4.4

1. (a) $y'' = 110x^9$

(b) $y'' = 210x^5 - 36x$

(c) $y'' = \frac{18}{(3x+2)^3}$

(d) $y'' = \frac{8}{(1-2x)^3}$

(e) $y'' = -\frac{1}{4}(x+5)^{-\frac{3}{2}}$

(f) $y'' = -\frac{2}{9}(x-4)^{-\frac{5}{3}}$

(g) $y'' = 3e^{\sqrt{3}x}$

(h) $y'' = -\frac{16}{(4x+5)^2}$

(i) $y'' = -\pi^2 \sin \pi x$

(j) $y'' = -\frac{1}{3} \cos \frac{1}{\sqrt{3}}x$

3. (a) $y^{(n)} = (-1)^n n! (x+2)^{-(n+1)}$

(b) $y^{(n)} = (-1)^n n! 3^n (3x-1)^{-(n+1)}$

(c) $y^{(n)} = 10^n e^{10x}$

(d) $y^{(n)} = (-1)^{n-1} (n-1)! 2^n (2x+3)^{-n}$

(e) $y^{(n)} = \begin{cases} \sin x, & n = 4k \\ \cos x, & n = 4k+1 \\ -\sin x, & n = 4k+2 \\ -\cos x, & n = 4k+3 \end{cases}$

(f) $y^{(n)} = \begin{cases} 2^n \cos 2x, & n = 4k \\ -2^n \sin 2x, & n = 4k+1 \\ -2^n \cos 2x, & n = 4k+2 \\ 2^n \sin 2x, & n = 4k+3 \end{cases}$

(g) $y^{(n)} = \begin{cases} 3^n \cosh 3x, & n \text{ is odd} \\ 3^n \sinh 3x, & n \text{ is even} \end{cases}$

(h) $y^{(n)} = \begin{cases} 4^n \sinh 4x, & n \text{ is odd} \\ 4^n \cosh 4x, & n \text{ is even} \end{cases}$

5. (a) $\frac{d^2 y}{dx^2} = \frac{3}{4t} \quad (\forall t \neq 0)$

(b) $\frac{d^2 y}{dx^2} = -\frac{1}{4t^3}$

(c) $\frac{d^2 y}{dx^2} = -\frac{1}{2s^4} \quad (\forall s \neq 0)$

(d) $\frac{d^2 y}{dx^2} = \frac{3}{4} s^{-\frac{1}{6}}$

(e) $\frac{d^2 y}{dx^2} = -\sec^3 \theta$

(f) $\frac{d^2 y}{dx^2} = -\csc^3 \theta$