CHAPTER 01 연습문제 정답

1.1

(a) $A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$

(b) $x + y + z = 1$

1.2

$A + B = \begin{bmatrix} 5 & 5 \\ 2 & 4 \\ 4 & 13 \end{bmatrix}, \quad -2A = \begin{bmatrix} -8 & -10 \\ -4 & -6 \\ -2 & -18 \end{bmatrix}, \quad 3A - B = \begin{bmatrix} 11 & 15 \\ 6 & 8 \\ 0 & 23 \end{bmatrix}$

1.3

(a) $AB = \begin{bmatrix} 3 & 9 & -1 \\ -2 & 2 & -2 \end{bmatrix}, \quad BC = \begin{bmatrix} 9 \\ 10 \end{bmatrix}$

(b) $AB = \begin{bmatrix} -8 & 7 \\ 37 & 8 \end{bmatrix}, \quad BA = \begin{bmatrix} -9 & 22 \\ 19 & 9 \end{bmatrix}$

1.4
생략

1.5
생략

1.6

(a) $x_1 = \dfrac{-22}{7}, \; x_2 = \dfrac{39}{7}$

(b) $x_1 = 0, \; x_2 = -3, \; x_3 = -5$

1.7
(a) 해가 무수히 많다.
(b) 해가 없다.

1.8

(a) $x_1 = \dfrac{155}{7}$, $x_2 = \dfrac{27}{7}$, $x_3 = -\dfrac{132}{35}$

(b) $x_1 = \dfrac{97}{53}$, $x_2 = \dfrac{942}{53}$, $x_3 = -\dfrac{61}{53}$, $x_4 = -\dfrac{272}{53}$

1.9

사자 : 5마리, 학 : 2마리

1.10

(a) $\begin{bmatrix} -5 & 4 & -3 \\ 5 & -16 & -18 \end{bmatrix}$

(b) $\begin{bmatrix} 28 \end{bmatrix}$

1.11

(a) 거짓

(b) 거짓

CHAPTER 02 연습문제 정답

2.1
A : 삼각행렬, B : 대칭행렬, C : 대각행렬, D : 반대칭행렬

2.2
생략

2.3
$x = y = 1$ 또는 $x = y = 0$

2.4
생략

2.5
(a) $A^{-1} = \dfrac{1}{16} \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$

(b) $B^{-1} = \dfrac{1}{33} \begin{bmatrix} -6 & 5 \\ 9 & -2 \end{bmatrix}$

2.6
생략

2.7
생략

2.8
(a) 생략

(b) A의 행렬식은 -81이다.

2.9

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

2.10

(a) $X = \dfrac{1}{2}\begin{bmatrix} 26 & -16 \\ 23 & 16 \end{bmatrix}$

(b) $X = \begin{bmatrix} 0 & -6 \\ 3 & 9 \end{bmatrix}$

(c) $X = -\dfrac{1}{25}\begin{bmatrix} -19 & 0 \\ 6 & 0 \\ -35 & -75 \end{bmatrix}$

2.11

(a) $x_1 = -\dfrac{13}{4},\ x_2 = \dfrac{31}{4}$

(b) $x_1 = 0,\ x_2 = -3,\ x_3 = -5$

2.12

(a) $x_1 = -\dfrac{1}{35},\ x_2 = \dfrac{135}{35},\ x_3 = -\dfrac{132}{35}$

(b) $x_1 = 1,\ x_2 = 2,\ x_3 = 3,\ x_4 = 4$

2.13

(a) $tr(AB) = 4$
(b) $tr(BA) = 4$

2.14
생략

2.15
생략

2.16
생략

CHAPTER 03 연습문제 정답

3.1
생략

3.2
생략

3.3
생략

3.4
(a) 선형독립
(b) 선형종속

3.5
생략

3.6
H의 기저는 $\{\boldsymbol{x}_1, \boldsymbol{x}_2\}$이다.

3.7
$\dfrac{1}{9}(2, -2, 1)$

3.8
(a) $\theta = \cos^{-1}\left(\dfrac{1}{14}\right)$

(b) $\theta = \cos^{-1}\left(\dfrac{6}{\sqrt{78}}\right)$

3.9
$x = 1$

3.10

$2 - 2\sqrt{2}$

3.11

생략

3.12

$\nabla f = \begin{bmatrix} 2x + 7y \\ 8y + 7x \end{bmatrix}, \quad \nabla f_{(2,5)} = \begin{bmatrix} 39 \\ 51 \end{bmatrix}$

3.13

$\nabla f = \begin{bmatrix} 2x + y^2 \\ 2xy + z \\ y + 1 \end{bmatrix}, \quad \nabla f_{(1,2,3)} = \begin{bmatrix} 6 \\ 7 \\ 3 \end{bmatrix}$

$H(f) = \begin{bmatrix} 2 & 2y & 0 \\ 2y & 2x & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad H(f)_{(1,2,3)} = \begin{bmatrix} 2 & 4 & 0 \\ 4 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

CHAPTER 04 연습문제 정답

4.1
(a) L은 선형변환이다.
(b) L은 선형변환이 아니다.

4.2
(a) $\begin{bmatrix} 1 & -3 \\ 2 & 1 \\ 1 & -2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 & -1 & 5 \\ 4 & 1 & 2 & 5 \end{bmatrix}$

4.3
L은 일대일 변환이 아니고 위로의 변환이다.

4.4
생략

4.5
$P\left(2,\, 3\sqrt{2}\,\right)$

4.6
$\begin{bmatrix} 9 \\ 7 \\ 22 \end{bmatrix}$

4.7
(a) 랭크 : 2
(b) 랭크 : 3

4.8
(a) 랭크 : 3, 퇴화차수 : 1
(b) 랭크 : 2, 퇴화차수 : 1

4.9
차원 = 3

4.10
랭크 : 3,   영공간 $= \mathrm{span}\left\{ \begin{bmatrix} 4 \\ -2 \\ -1 \\ 1 \end{bmatrix} \right\}$

4.11
(a) 직교연산자이다.
(b) 직교연산자이다.
(c) 직교연산자이다.

4.12
생략

CHAPTER 05 연습문제 정답

5.1
(a) $\lambda = -2,\ 10$

(b) $\lambda = 3,\ 4$

5.2
(a) $(\lambda - 3)(\lambda + 1)$

(b) $(\lambda - 1)(\lambda - 2)(\lambda - 8)$

5.3

ⅰ) $\lambda_1 = 7$; $E_1 = \mathrm{span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

ⅱ) $\lambda_2 = \dfrac{-\sqrt{109} + 5}{2}$; $E_2 = \mathrm{span}\left\{ \begin{bmatrix} \dfrac{-\sqrt{109} - 3}{10} \\ 1 \\ 0 \end{bmatrix} \right\}$

ⅲ) $\lambda_3 = \dfrac{\sqrt{109} + 5}{2}$; $E_3 = \mathrm{span}\left\{ \begin{bmatrix} \dfrac{\sqrt{109} - 3}{10} \\ 1 \\ 0 \end{bmatrix} \right\}$

5.4
특성다항식 : $(\lambda + 2)(\lambda - 3)(\lambda - 5)$

ⅰ) $\lambda_1 = -2$; $E_1 = \mathrm{span}\left\{ \begin{bmatrix} -35 \\ 12 \\ 19 \end{bmatrix} \right\}$

ⅱ) $\lambda_2 = 3$; $E_2 = \mathrm{span}\left\{ \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \right\}$

ⅲ) $\lambda_3 = 5$; $E_3 = \mathrm{span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

5.5

(a) i) $\lambda_1 = 2\sqrt{3} - 1$; $E_1 = \text{span}\left\{\begin{bmatrix} \sqrt{3}-1 \\ 1 \end{bmatrix}\right\}$

　　ii) $\lambda_2 = -2\sqrt{3} - 1$; $E_2 = \text{span}\left\{\begin{bmatrix} -\sqrt{3}-1 \\ 1 \end{bmatrix}\right\}$

(b) i) $\lambda_1 = -1$; $E_1 = \text{span}\left\{\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right\}$

　　ii) $\lambda_2 = 2$; $E_2 = \text{span}\left\{\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}\right\}$

5.6

(a) -18

(b) 5

(c) 7

5.7

[파이썬 코드를 이용한 실행 결과]
A^20 =
 [[24157816   -87403800   63245985]
 [63245985 -228826124   165580140]
 [165580140 -599074575   433494436]]

5.8

$\lambda = k,\ (3+\sqrt{5})k,\ (3-\sqrt{5})k$

5.9

(a) $\lambda^2 - 5\lambda + 11$

(b) $\lambda^3 - \lambda^2 - 15\lambda + 15$

5.10

(a) 특성다항식 : $(\lambda - 1)(\lambda - 2)^2$

     i) $\lambda_1 = 1$, 고유벡터 $= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

     ii) $\lambda_2 = 2$, 고유벡터 $= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

(b) $A^{10} = \begin{bmatrix} 1 & 0 & 1003 \\ 0 & 1004 & 0 \\ 0 & 0 & 1004 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1 & 0 & -\dfrac{1}{2} \\ 0 & \dfrac{1}{2} & 0 \\ 0 & 0 & \dfrac{1}{2} \end{bmatrix}$

5.11

$tr(A) = 1$, $\det(A) = 18$

5.12

생략

CHAPTER 06 연습문제 정답

6.1

(a) $\begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 0 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 & -1 \\ 0 & -2 & 2 \\ -1 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & -1 \\ 0 & -2 & 2 \\ 0 & 0 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 1 & 0 & 1 \\ -2 & 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -3 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & -4 & 1 \end{bmatrix}$

6.2

생략

6.3

$\begin{bmatrix} \dfrac{1}{2} \\ 0 \\ 0 \\ -4 \end{bmatrix} + \begin{bmatrix} -\dfrac{3}{2} \\ 2 \\ 1 \\ 4 \end{bmatrix} t$ (t는 임의의 상수)

6.4

(a) $x_1 = -\dfrac{6}{5}$, $x_2 = \dfrac{1}{5}$, $x_3 = \dfrac{3}{5}$

(b) $x_1 = -1$, $x_2 = -1$, $x_3 = -1$

6.5

$x_1 = \dfrac{5}{6}$, $x_2 = -\dfrac{1}{3}$, $x_3 = -\dfrac{1}{2}$

6.6

$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$

6.7

(a)

$$A = \begin{bmatrix} \dfrac{7\sqrt{5}}{20} & \dfrac{4\sqrt{5}}{5} \\[3mm] \dfrac{2\sqrt{5}}{20} & \dfrac{4\sqrt{5}}{5} \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \dfrac{2}{\sqrt{5}} & \dfrac{1}{\sqrt{5}} \\[3mm] -\dfrac{1}{\sqrt{5}} & \dfrac{2}{\sqrt{5}} \end{bmatrix}$$

[파이썬 코드를 이용한 실행 결과]

A =
 [[4 6]
 [0 4]]

U=
 [[0.89442719 -0.4472136]
 [0.4472136   0.89442719]]

Sigma =
 [[8. 0.]
 [0. 2.]]

V^T =
 [[0.4472136   0.89442719]
 [-0.89442719   0.4472136]]

(b)

$$A = \begin{bmatrix} 1 & \dfrac{1}{\sqrt{2}} \\[3mm] 1 & \dfrac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 2\sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\[3mm] -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}$$

[파이썬 코드를 이용한 실행 결과]

A =
 [[2 2]
 [2 2]]

U=
 [[-0.70710678 -0.70710678]
 [-0.70710678   0.70710678]]

Sigma =
 [[4. 0.]
 [0. 0.]]

V^T =
 [[-0.70710678 -0.70710678]
 [-0.70710678 0.70710678]]

(c)
[파이썬 코드를 이용한 실행 결과]
A =
 [[1 1]
 [0 1]
 [1 0]]

U=
 [[-8.16496581e-01 -1.85577521e-16 -5.77350269e-01]
 [-4.08248290e-01 -7.07106781e-01 5.77350269e-01]
 [-4.08248290e-01 7.07106781e-01 5.77350269e-01]]

Sigma =
 [[1.73205081 0.   ]
 [0.   1.   ]
 [0.   0.   ]]

V^T =
 [[-0.70710678 -0.70710678]
 [0.70710678 -0.70710678]]

(d)
[파이썬 코드를 이용한 실행 결과]
A =
 [[-3 -1 2]
 [2 1 -2]]

U=
 [[-0.7815437 0.6238505]
 [0.6238505 0.7815437]]

Sigma =
 [[4.77289369 0.   0.   ]
 [0.   0.46849315 0.   ]]

V^T =
 [[7.52652862e-01 2.94453279e-01 -5.88906559e-01]
 [-6.58417549e-01 3.36596593e-01 -6.73193185e-01]
 [1.50028633e-16 8.94427191e-01 4.47213595e-01]]

(e)

$$A = \begin{bmatrix} 0 & \dfrac{\sqrt{5}}{2} & 0 \\ -\dfrac{2}{5} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2\sqrt{5} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[파이썬 코드를 이용한 실행 결과]
A =
 [[0 5 0]
 [-2 0 4]
 [0 0 0]]

U=
 [[1. 0. 0.]
 [0. 1. 0.]
 [0. 0. 1.]]

Sigma =
 [[5.   0.   0.   ]
 [0.   4.47213595 0.   ]
 [0.   0.   0.   ]]

V^T =
 [[-0.   1.   0.   ]
 [-0.4472136   0.   0.89442719]
 [-0.89442719 0.   -0.4472136]]

6.8
(a) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} \dfrac{2}{\sqrt{6}} & 0 \\ \dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{6}} & -\dfrac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{3} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{1}{\sqrt{2}} & 0 & \dfrac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\dfrac{1}{\sqrt{2}} & 0 & \dfrac{1}{\sqrt{2}} \end{bmatrix}$

6.9

(a) $A^{\mathrm{T}}A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$, $AA^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

i) $A^{\mathrm{T}}A$의 특잇값 분해 결과

[파이썬 코드를 이용한 실행 결과]

A =
 [[2 1]
 [1 1]]

U=
 [[-0.85065081 -0.52573111]
 [-0.52573111 0.85065081]]

sigma=
 [[2.61803399 0.   ]
 [0.   0.38196601]]

V^T =
 [[-0.85065081 -0.52573111]
 [-0.52573111 0.85065081]]

ii) AA^{T}의 특잇값 분해 결과

$$AA^T = \begin{bmatrix} -\frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}^T$$

(b) $A^{\mathrm{T}}A = \begin{bmatrix} 13 & 0 & 6 \\ 0 & 0 & 0 \\ 6 & 0 & 4 \end{bmatrix}$, $AA^{\mathrm{T}} = \begin{bmatrix} 9 & 6 \\ 6 & 4 \end{bmatrix}$

i) $A^{\mathrm{T}}A$의 특잇값 분해 결과

$$A^T A = \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \end{bmatrix} \begin{bmatrix} 16 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \end{bmatrix}^T$$

ii) AA^{T}의 특잇값 분해 결과

$$AA^T = \begin{bmatrix} \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \\ \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{bmatrix} \begin{bmatrix} 13 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \\ \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{bmatrix}^T$$

6.10

[파이썬 코드를 이용한 실행 결과]

A =
 [[-3 -1 2]
 [2 1 -2]]

Pseudo inverse of A =
 [[-1. -1.]
 [0.4 0.6]
 [-0.8 -1.2]]

CHAPTER 07 연습문제 정답

7.1

(a) $f'(x) = 77x^{76} - 2x + 1$

(b) $f'(x) = 4x^3 - 21x^2$

(c) $f'(x) = 5x^4 + 2e^{2x}$

(d) $f'(x) = \sqrt{3}$

(e) $f'(x) = -\dfrac{1}{2x\sqrt{x}} + \dfrac{1}{5\sqrt[5]{x^4}}$

7.2

(a) $f'(x) = 3\cos x + 5\sin x$

(b) $f'(x) = 2x\sin x + x^2\cos x$

(c) $f'(x) = \dfrac{\cos x}{(1+\cos x)^2}$

(d) $f'(x) = \dfrac{1 - x\sec x}{\sec x + \tan x}$

(e) $f'(x) = e^x(\cos x - \sin x)$

7.3

(a) $y = 4x + 4$

(b) $y = 4x + 3$

(c) $y = \dfrac{5}{2}x - \dfrac{3}{2}$

(d) $y = x + 1$

(e) $y = \pi$

7.4

(a) 증명 생략, $c = 1$

(b) 증명 생략, $c = \dfrac{\pi}{2}$

(c) 증명 생략, $c = 1$

7.5

(a) 증명 생략, $c = \dfrac{3}{2}$

(b) 증명 생략, $c = 0,\ \pi,\ 2\pi$

(c) 증명 생략, $c = 4$

7.6

(a) 증명 생략, 극한값 $= \dfrac{1}{6}$

(b) 증명 생략, 극한값 $= 2$

(c) 증명 생략, 로피탈 정리 적용 불가능

(d) 증명 생략, 극한값 $= 0$

(e) 증명 생략, 극한값 $= \dfrac{\log 7 - \log 5}{\log 4 - \log 3}$

7.7

생략

7.8

(a) $f'(x) = 10(x^7 + 3x^3 - 5x)^9(7x^6 + 9x^2 - 5)$

(b) $f'(x) = 10(5x + 9)(x^2 - x + 7)^9(11x^2 + 12x - 2)$

(c) $f'(x) = -\sin x \cos(\cos x)$

(d) $f'(x) = (\sin 2x + 2x \cos 2x)e^{x\sin 2x} + 3x^2 2^{x^3}\log 2$

(e) $f'(x) = 2x(x + 1)e^{2x} + \dfrac{7^{\sqrt{x}}\log 7}{2\sqrt{x}}$

7.9

생략

7.10

$a = -7,\ b = -12$

CHAPTER 08 연습문제 정답

8.1
생략

8.2
$-2\cos x + C$

8.3
$\dfrac{1}{545}(x^5 + 5x + 1)^{109} + C$

8.4
$\dfrac{\cos^5 x}{5} - \dfrac{\cos^3 x}{3}$

8.5
$2e^{\sqrt{x}} + C$

8.6
(a) $\dfrac{1}{4}x^2(2\log x - 1)$

(b) $\dfrac{1}{2}e^x(\sin x + \cos x)$

8.7
(a) 54

(b) $\dfrac{10}{3}$

(c) $\log\dfrac{19}{7}$

(d) 0

8.8
생략

8.9

(a) 102

(b) 1

8.10

(a) $\dfrac{3\pi}{2}$

(b) $\dfrac{1}{54}\left(109\sqrt{109} - 37\sqrt{37}\right)$

8.11

(a) π

(b) π

8.12

(a) $\dfrac{4}{3}\sqrt{2}\left(5\sqrt{5} - 1\right)\pi$

(b) $\dfrac{17\sqrt{17} - 27}{6}\pi$

CHAPTER 09 연습문제 정답

9.1

(a) 정의역 : $\mathbb{R}^2$, 치역 : $\mathbb{R}$

(b) 정의역 : $D = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 < 1\}$, 치역 : $(-\infty, 0]$

9.2

(a) $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = k\}$

(b) $\{(x, y) \in \mathbb{R}^2 \mid y - x^2 = k\}$
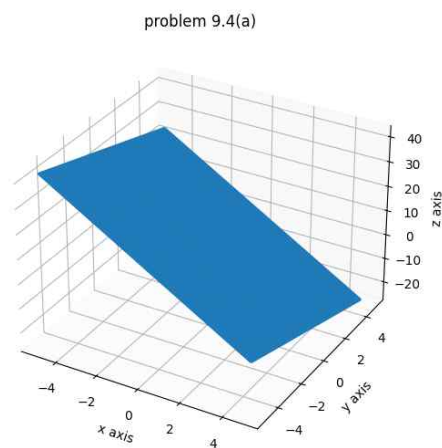
9.3

$$\frac{\partial z}{\partial u} = 3\left((v + e^u)^2 + u^2 v\right)^2 \left(2uv + 2e^u(v + e^u)\right) + 4u^3 v^2 (v + e^u) + u^4 v^2 e^u$$

$$\frac{\partial z}{\partial v} = 3\left((v + e^u)^2 + u^2 v\right)^2 \left(2v + 2e^u + u^2\right) + 2u^4 v(v + e^u) + u^4 v^2$$

9.4

[파이썬 코드를 이용한 실행 결과]

(a)

problem 9.4(a)
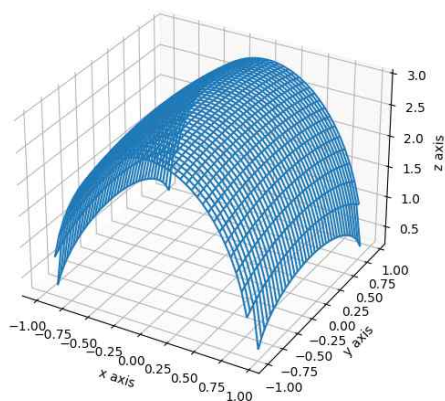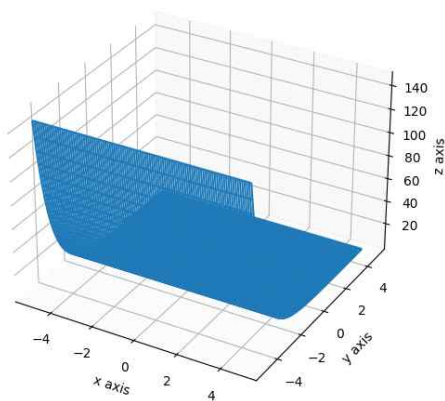
(b)

problem 9.4(b)

(c)

problem 9.4(c)

9.5

(a) 2

(b) 0

(c) $-\dfrac{11}{633}$

9.6

(a) $\mathbb{R}^2$

(b) $\{(x, y) \in \mathbb{R}^2 \mid y \neq 0\}$

(c) $\{(x, y) \in \mathbb{R}^2 \mid x + 3y \geq 0\}$

(d) $\{(x, y) \in \mathbb{R}^2 \mid x \neq y\}$

9.7

(a) $\dfrac{\partial f}{\partial x} = -4x,\ \dfrac{\partial f}{\partial y} = 6y^5 - 4y$

(b) $\dfrac{\partial f}{\partial x} = \sec^2(x+y),\ \dfrac{\partial f}{\partial y} = \sec^2(x+y)$

(c) $\dfrac{\partial f}{\partial x} = \dfrac{e^y}{(x+y^2)^2},\ \dfrac{\partial f}{\partial y} = \dfrac{e^y(x+(y-2)y)}{(x+y^2)^2}$

(d) $\dfrac{\partial f}{\partial x} = \dfrac{x}{y\sqrt{x^2+y^2}+x^2+y^2},\ \dfrac{\partial f}{\partial y} = \dfrac{1}{\sqrt{x^2+y^2}}$

9.8

(a) $\left(3x^2y^6 + 5x^4y^4,\ 6x^3y^5 + 4x^5y^3\right)$

(b) $\left(3\cos(3x+7y),\ 7\cos(3x+7y)\right)$

(c) $\left(e^{-y},\ -xe^{-y}\right)$

(d) $\left(\dfrac{1}{x+3y},\ \dfrac{3}{x+3y}\right)$

9.9

(a) $\begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -2 & 4 \\ -2 & 2 & 4 \\ 4 & 4 & 4 \end{bmatrix}$

9.10

(a) 부정부호행렬
(b) 음의 준정부호행렬
(c) 양의 준정부호행렬

CHAPTER 10 연습문제 정답

10.1

(a) $\dfrac{23}{40}$

(b) $\dfrac{1}{4}$

(c) $\dfrac{5}{8}$

10.2

(a) 독립이 아니다.
(b) 독립이다.
(c) 독립이 아니다.
(d) 독립이 아니다.
(e) 독립이 아니다.

10.3

$p = \dfrac{1}{6}$

10.4

(a) 0.525
(b) 0.175

10.5

0.016

10.6

0.036

10.7

(a) $f_X(x) = \begin{cases} \dfrac{4^x e^{-4}}{x!} & , \ x = 0, 1, 2, \cdots \\ 0 & , \ \text{그 외} \end{cases}$

(b) 0.762

(c) $\mathrm{E}(X) = 4, \ \mathrm{V}(X) = 4$

10.8

680명

10.9

$\mu_Y = a\mu_X + b, \ \sigma_Y^2 = (a\sigma_X)^2, \ \sigma_Y = a\sigma_X$

10.10

생략

CHAPTER 11 연습문제 정답

11.1
생략

11.2
생략

11.3
(a) 4

(b) $\mathrm{E}(Y) = -5, \ \mathrm{V}(Y) = 36$

11.4
생략

11.5
생략

11.6
(a) $\mathrm{E}(X) = \dfrac{2}{3}, \ \mathrm{V}(X) = \dfrac{2}{9}$

(b) $\mathrm{E}(Y) = \dfrac{3}{4}, \ \mathrm{V}(Y) = \dfrac{11}{16}$

(c) $\mathrm{E}(XY) = \dfrac{13}{24}$

(d) $\mathrm{Cov}(X, \ Y) = \dfrac{1}{24}$

(e) $\mathrm{V}(X + Y) = \dfrac{143}{144}$

11.7

(a) $k = 27$

(b)

Y \ X	0	1	2	$P(Y = y_j)$
0	0	$\dfrac{1}{27}$	$\dfrac{2}{27}$	$\dfrac{1}{9}$
1	$\dfrac{2}{27}$	$\dfrac{3}{27}$	$\dfrac{4}{27}$	$\dfrac{1}{3}$
2	$\dfrac{4}{27}$	$\dfrac{5}{27}$	$\dfrac{6}{27}$	$\dfrac{5}{9}$
$P(X = x_i)$	$\dfrac{2}{9}$	$\dfrac{1}{3}$	$\dfrac{4}{9}$	1

(c) $\mathrm{Corr}(X,\ Y) = 0.184$

11.8

$y = 1.29 + 1.40x$

11.9

$y = 1.83 + 0.86x + 0.17y$

11.10

$y = 5.20 + 0.40x$, $y(9) = 8.80$

11.11

$y = -0.76 + 0.11x$, $y(30) = 2.54$

CHAPTER 12 연습문제 정답

12.1

$\dfrac{2}{3}$

12.2

혼동행렬은 다음과 같다.

		예측된 클래스	
		양성	음성
실제 클래스	양성	11	4
	음성	4	0

$(\text{정밀도}) = \dfrac{11}{15},\quad (\text{재현율}) = \dfrac{11}{15},\quad (\text{F1 점수}) = \dfrac{11}{30}$

12.3

$(\text{평균절대오차}) = 1,\quad (\text{평균제곱오차}) = 1.33$

$(\text{평균제곱근오차}) = 1.15,\quad (R^2 \ \text{점수}) = 0.80$

12.4

[파이썬 코드를 이용한 실행 결과]

입력 데이터의 라벨 = ['배']

12.5

i) 엔트로피를 이용한 정보 이득 함수의 값 $= 0.24$
ii) 지니 불순도를 이용한 정보 이득 함수의 값 $= 0.13$

12.6

[파이썬 코드를 이용한 실행 결과]

투영행렬 W =
 [[0.59140322]
 [-0.57633338]
 [-0.56398766]]

12.7

$y = 1.25 + 1.37x + 0.01x^2$

12.8

$\mu_1 = (-1, 0),\ \mu_2 = (1.75,\ 1.75)$

CHAPTER 13 연습문제 정답

13.1

(a) $\tau(\boldsymbol{w}\boldsymbol{x}_1^{\mathsf{T}}) = -1$, $\sigma(\boldsymbol{w}\boldsymbol{x}_1^{\mathsf{T}}) \fallingdotseq 0.01$, $\mathrm{ReLU}(\boldsymbol{w}\boldsymbol{x}_1^{\mathsf{T}}) = 0$

(b) $\tau(\boldsymbol{w}\boldsymbol{x}_2^{\mathsf{T}}) = 1$, $\sigma(\boldsymbol{w}\boldsymbol{x}_2^{\mathsf{T}}) \fallingdotseq 1.00$, $\mathrm{ReLU}(\boldsymbol{w}\boldsymbol{x}_2^{\mathsf{T}}) = 13$

13.2

$$\sigma(\boldsymbol{w}\boldsymbol{x}_0^{\mathsf{T}}) = \sigma(2) = \frac{1}{1+e^{-2}} \fallingdotseq 0.88$$

$$\mathrm{ReLU}(\boldsymbol{w}\boldsymbol{x}_0^{\mathsf{T}}) = \mathrm{ReLU}(2) = 2$$

13.3

(a) $\tau(\boldsymbol{w}\boldsymbol{x}_1^{\mathsf{T}}) = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$, $\sigma(\boldsymbol{w}\boldsymbol{x}_1^{\mathsf{T}}) \fallingdotseq (0.01,\ 1.00,\ 0.01)$, $\mathrm{ReLU}(\boldsymbol{w}\boldsymbol{x}_1^{\mathsf{T}}) = (0,\ 5,\ 0)$

(b) $\tau(\boldsymbol{w}\boldsymbol{x}_2^{\mathsf{T}}) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\sigma(\boldsymbol{w}\boldsymbol{x}_2^{\mathsf{T}}) \fallingdotseq (1.00,\ 0.88,\ 1.00)$, $\mathrm{ReLU}(\boldsymbol{w}\boldsymbol{x}_2^{\mathsf{T}}) = (13,\ 2,\ 30)$

13.4

$(0.00,\ 0.01,\ 0.99)$

13.5

$\boldsymbol{c} = (\cdots,\ 0,\ -9,\ 22,\ 0,\ -8,\ 3,\ 31,\ 0,\ -5,\ 0,\ \cdots)$

13.6

$n_{out}^2 = 168$

13.7

$C = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$

13.8

$$\begin{bmatrix} \dfrac{1}{2} \\ \dfrac{1}{2} \end{bmatrix}$$