

7장_기초 회귀분석 연습문제 풀이

01.

$$\bar{x} = \frac{65}{11} \approx 5.91, \quad \bar{y} = \frac{45}{11} \approx 4.09$$

$$S_{xy} = \frac{1}{10} [258 - (11)(5.91)(4.09)] \approx -0.7909, \quad S_{xx} = \frac{1}{10} [401 - (11)(5.91)^2] \approx 1.691$$

$$b = -\frac{0.7909}{1.691} \approx -0.4677, \quad a = 4.09 - (-0.4677)(5.91) \approx 6.8541, \quad \hat{y} = 6.8541 - 0.4677x$$

03.

번호	x	y	x^2	xy
1	1	2	1	2
2	2	5	4	10
3	3	6	9	18
4	3	4	9	12
5	4	8	16	32
6	8	9	64	72
7	14	15	196	210
합계	35	49	299	356

$$\sum x_i = 35, \quad \sum y_i = 49, \quad \sum x_i^2 = 299, \quad \sum x_i y_i = 356$$

$$\bar{x} = \frac{35}{7} = 5, \quad \bar{y} = \frac{49}{7} = 7$$

$$S_{xy} = \frac{1}{6} [356 - (7)(5)(7)] = 18.5, \quad S_{xx} = \frac{1}{6} [299 - (7)(5)^2] \approx 20.667$$

$$b = \frac{18.5}{20.667} \approx 0.8952, \quad a = 7 - (0.895)(5) = 2.5242, \quad \hat{y} = 2.5242 + 0.8952x$$

05.

번호	x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
1	2	17	-9	4	81	16	-36
2	4	13	-7	0	49	0	0
3	6	18	-5	5	25	25	-25
4	8	16	-3	3	9	9	-9
5	10	16	-1	3	1	9	-3
6	12	13	1	0	1	0	0
7	14	14	3	1	9	1	3
8	16	10	5	-3	25	9	-15
9	18	8	7	-5	49	25	-35
10	20	5	9	-8	81	64	-72
합계	110	130	0	0	330	158	-192

$$\bar{x} = \frac{110}{10} = 11, \quad \bar{y} = \frac{130}{10} = 13, \quad S_{xy} = \frac{1}{9} \sum (x_i - \bar{x})(y_i - \bar{y}) = -\frac{192}{9} \approx -21.3333,$$

$$S_{xx} = \frac{1}{9} \sum (x_i - \bar{x})^2 = \frac{330}{9} \approx 36.6667, \quad S_{yy} = \frac{1}{9} \sum (y_i - \bar{y})^2 = \frac{158}{9} \approx 17.5556$$

$$(a) \sigma_x = \sqrt{36.6667} = 6.0553, \quad \sigma_y = \sqrt{17.5556} = 4.1899$$

$$\rho = \frac{S_{xy}}{\sigma_x \sigma_y} = -\frac{21.3333}{(6.0553)(4.1899)} \approx -0.84085, \text{ 음의 상관관계}$$

$$(b) b = -\frac{21.3333}{36.6667} \approx -0.58182, \quad a = 13 - (-0.58182)(11) = 19.4, \quad \hat{y} = 19.4 - 0.58182x$$

$$(c) SSE = (n-1)(S_{yy} - bS_{xy}) = (9)[17.5556 - (-0.58182)(-21.3333)] \approx 46.29$$

$$(d) SST = \sum (y_i - \bar{y})^2 = 158$$

$$(e) SSR = SST - SSE = 158 - 46.29 = 111.71$$

$$(f) R^2 = \frac{SSR}{SST} = \frac{111.71}{158} \approx 0.70702$$

07.

번호	x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
1	3	4	-1	0	1	0	0
2	5	4	1	0	1	0	0
3	4	5	0	1	0	1	0
4	3	3	-1	-1	1	1	1
5	5	4	1	0	1	0	0
6	5	7	1	3	1	9	3
7	2	3	-2	-1	4	1	2
8	6	3	2	-1	4	1	-2
9	4	3	0	-1	0	1	0
10	3	4	-1	0	1	0	0
합계	40	40	0	0	14	14	4

(a) 추정 회귀방정식

$$\bar{x} = \frac{40}{10} = 4, \quad \bar{y} = \frac{40}{10} = 4, \quad S_{xy} = \frac{1}{9} \sum (x_i - \bar{x})(y_i - \bar{y}) = \frac{4}{9} \approx 0.44444,$$

$$S_{xx} = \frac{1}{9} \sum (x_i - \bar{x})^2 = \frac{14}{9} \approx 1.55556, \quad S_{yy} = \frac{1}{9} \sum (y_i - \bar{y})^2 = \frac{14}{9} \approx 1.55556$$

$$b = \frac{0.44444}{1.55556} \approx 0.2857, \quad a = 4 - (0.2857)(4) = 2.8572, \quad \hat{y} = 2.8572 + 0.2857x$$

(b) 스타트업의 다양성 수준과 창업성과의 관계

$$\sigma_x = \sqrt{1.5556} = 1.2472, \quad \sigma_y = \sqrt{1.5556} = 1.2472$$

$$\rho = \frac{S_{xy}}{\sigma_x \sigma_y} = \frac{0.4444}{(1.2472)^2} \approx 0.2857, \quad \text{거의 상관관계가 없다.}$$

(c) 기대 성과

$$\hat{y}_{x=8} = 2.8572 + 0.2857(8) \approx 5.1428$$

(d) 평균제곱오차

$$SSE = (n-1)(S_{yy} - b S_{xy}) = (9)[1.55556 - (0.2857)(0.44444)] \approx 12.8573$$

$$MSE = \frac{SSE}{n-2} = \frac{12.8573}{8} \approx 1.6072$$

(e) 결정계수

$$SST = \sum (y_i - \bar{y})^2 = 14$$

$$SSR = SST - SSE = 14 - 12.8573 = 1.1427$$

$$R^2 = \frac{SSR}{SST} = \frac{1.1427}{14} \approx 0.0816$$

09.

번호	x	y	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})^2$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$
1	3	5	-0.5	1	0.25	1	-0.5
2	5	2	1.5	-2	2.25	4	-3
3	4	3	0.5	-1	0.25	1	-0.5
4	3	3	-0.5	-1	0.25	1	0.5
5	5	5	1.5	1	2.25	1	1.5
6	5	3	1.5	-1	2.25	1	-1.5
7	2	3	-1.5	-1	2.25	1	1.5
8	4	5	0.5	1	0.25	1	0.5
9	1	4	-2.5	0	6.25	0	0
10	3	7	-0.5	3	0.25	9	-1.5
합계	35	40	0	0	16.5	20	-3

(a) 추정 회귀방정식

$$\bar{x} = \frac{35}{10} = 3.5, \quad \bar{y} = \frac{40}{10} = 4, \quad S_{xy} = \frac{1}{9} \sum (x_i - \bar{x})(y_i - \bar{y}) = -\frac{3}{9} \approx -0.33333,$$

$$S_{xx} = \frac{1}{9} \sum (x_i - \bar{x})^2 = \frac{16.5}{9} \approx 1.83333, \quad S_{yy} = \frac{1}{9} \sum (y_i - \bar{y})^2 = \frac{20}{9} \approx 2.22222$$

$$b = -\frac{0.33333}{1.83333} \approx -0.18182, \quad a = 4 - (-0.18182)(3.5) = 4.63637, \quad \hat{y} = 4.63637 - 0.18182x$$

(b) 두 수준 사이의 관계

$$\sigma_x = \sqrt{1.8333} = 1.354, \quad \sigma_y = \sqrt{2.2222} = 1.4907$$

$$\rho = \frac{S_{xy}}{\sigma_x \sigma_y} = -\frac{0.33333}{(1.354)(1.4907)} \approx -0.165145, \text{ 거의 상관관계가 없다.}$$

(c) 창의성 수준이 7일 때 기대되는 기업의 혁신 수준

$$\hat{y}_{x=7} = 4.63637 - 0.18182(7) \approx 3.36363$$

(d) 평균제곱오차

$$SSE = (n-1)(S_{yy} - b S_{xy}) = (9)[2.22222 - (-0.18182)(-0.33333)] \approx 19.4545$$

$$MSE = \frac{SSE}{n-2} = \frac{19.4545}{8} \approx 2.4318$$

(e) 결정계수

$$SST = \sum (y_i - \bar{y})^2 = 20$$

$$SSR = SST - SSE = 20 - 19.4545 = 0.5455$$

$$R^2 = \frac{SSR}{SST} = \frac{0.5455}{20} \approx 0.027275$$