

[Chapter 6] 연습문제 정답

6.1

[풀이]

$$y = a_0 \sum_{m=0}^{\infty} \frac{(-ax)^m}{m!}$$

6.2

[풀이]

$$\therefore y = -2 + a_3 x^3$$

6.3

[풀이]

$$\therefore y = a_0 \left(1 - \frac{9x^2}{2!} + \frac{81x^4}{4!} + \dots \right) + a_1 \left(x - \frac{9x^3}{3!} + \frac{81x^5}{5!} + \dots \right)$$

6.4

[풀이]

$$\therefore y = a_0(1+x)$$

6.5

[풀이]

$$\therefore y = x^2 - x + a_0(1 - x + x^2 - x^3 + \dots)$$

6.6

[풀이]

$$\therefore y = a_0 \left(1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{3}{8}x^4 + \frac{3}{8}x^5 + \dots \right)$$

6.7

[풀이]

$$\therefore y = a_0 \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} x^m$$

6.8

[풀이]

$$\therefore y = a_0 \left(1 - \frac{1}{6}x^3 + \frac{1}{180}x^6 - \frac{1}{12960}x^9 - \dots \right) + a_1 \left(x - \frac{1}{12}x^4 + \frac{1}{504}x^7 - \frac{1}{45360}x^{10} - \dots \right)$$

6.9

[풀이]

$$\therefore y = a_0 \sum_{m=0}^{\infty} \frac{(-1)^m}{2^m m!} x^{2m} + a_1 \sum_{m=0}^{\infty} \frac{(-1)^m 2^m m!}{(2m+1)!} x^{2m+1}$$

6.10

[풀이]

$$\therefore y = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 \dots$$

6.11

[풀이]

$$\therefore a_2 = -\frac{n(n+1)}{2!}a_0$$

6.12

[풀이]

$$\therefore a_3 = -\frac{(n-1)(n+2)}{3!}a_1$$

6.13

[풀이]

$$\therefore a_{s+2} = -\frac{(n-s)(n+s+1)}{(s+2)(s+1)}a_s$$

6.14

[풀이]

$$\begin{aligned} \therefore y_1(x) &= 1 - \frac{n(n+1)}{2!}x^2 + \frac{(n-2)n(n+1)(n+3)}{4!}x^4 - \dots \\ y_2(x) &= x - \frac{(n-1)(n+2)}{3!}x^3 + \frac{(n-3)(n-1)(n+2)(n+4)}{5!}x^5 - \dots \end{aligned}$$

6.15

[풀이]

$$\begin{aligned} \therefore y_1(x) &= a_0 \left(x - \frac{6}{5}x^2 + \frac{6}{7}x^3 - \frac{4}{9}x^4 + \dots \right) \\ y_2(x) &= A_0 x^{-\frac{1}{2}} \end{aligned}$$

6.16

[풀이]

$$y_1(x) = a_0 \sum_{p=0}^{\infty} (-1)^p \left(\frac{3}{4}\right)^p \frac{x^{2p + \frac{5}{6}}}{p! \cdot 1 \cdot 4 \cdot 7 \cdots (3p+1)}$$

$$y_2(x) = A_0 \sum_{p=0}^{\infty} (-1)^p \left(\frac{3}{4}\right)^p \frac{x^{2p + \frac{1}{6}}}{p! \cdot 2 \cdot 5 \cdot 8 \cdots (3p-1)}$$

6.17

[풀이]

$$\therefore y_1(x) = 3a_0 x^2 \left(\frac{1}{3} - \frac{x^2}{2 \cdot 5} + \frac{x^4}{2! \cdot 2^2 \cdot 7} - \frac{x^6}{3! \cdot 2^3 \cdot 9} + \cdots \right)$$

6.18

[풀이]

$$\therefore y_1(x) = \frac{a_0}{x^2} \left(1 + \frac{(2x)^2}{3!} + \frac{(2x)^4}{5!} + \cdots \right)$$

6.19

[풀이]

$$\therefore y_1(x) = a_0 \left(1 + \frac{1}{2^2} x^2 + \frac{1}{4^2 \cdot 2^2} x^4 + \frac{1}{6^2 \cdot 4^2 \cdot 2^2} x^6 + \cdots \right)$$

6.20

[풀이]

$$\therefore y_1(x) = |x|^{r_1} \sum_{m=0}^{\infty} a_m x^m = x^2 a_0 = x^2$$

6.21

[풀이]

$$\therefore y_1(x) = a_0(1+x), \quad y_2(x) = A_0 x^{\frac{1}{2}}$$

6.22

[풀이]

$$\therefore y_1(x) = |x|^{r_1} \sum_{m=0}^{\infty} a_m x^m = x a_0 = x$$