

[Chapter 04] 연습문제 정답

4.1

$$[\text{풀이}] \quad \therefore y^{(4)} + (a+b+c)y''' + (ab+bc+ca)y'' + abcy' = 0$$

4.2

$$[\text{풀이}] \quad \therefore y''' + ay'' + b^2y' + ab^2y = 0$$

4.3

$$[\text{풀이}] \quad \therefore y^{(4)} + 2a^2y'' + a^4 = 0$$

4.4

$$[\text{풀이}] \quad \therefore y''' + (b+2c)y'' + (c^2+2bc)y' + bc^2y = 0$$

4.5

$$[\text{풀이}] \quad y''' - (a+b+c)y'' + (ab+bc+ca)y' - abcy = 0$$

$$y = c_1e^{ax} + c_2e^{bx} + c_3e^{cx}$$

4.6

$$[\text{풀이}] \quad y''' + (a+b)y'' + (ab+c)y' + cay = 0$$

$$y = c_1e^{-ax} + e^{-\frac{b}{2}x} \left(A \cos \frac{\sqrt{b^2-4ac}}{2} x + B \sin \frac{\sqrt{b^2-4ac}}{2} x \right)$$

4.7

$$[\text{풀이}] \quad y^{(4)} + (a^2+b^2)y'' + a^2b^2y = 0$$

$$y = A_1 \cos ax + B_1 \sin ax + A_2 \cos bx + B_2 \sin bx$$

4.8

$$[\text{풀이}] \quad y''' - 3cy'' + 3c^2y' - c^3y = 0$$

$$y = (c_1 + c_2x + c_3x^2)e^{cx}$$

4.9

$$[\text{풀이}] \quad y = c_1e^{ax} + (c_2 + c_3x)e^{bx} + e^{px}(A_1 \cos qx + B_1 \sin qx) + xe^{px}(A_2 \cos qx + B_2 \sin qx)$$

4.10

$$[\text{풀이}] \quad \therefore y = C_1 + C_2x + C_3x^2 + C_4x^3$$

4.11

$$[\text{풀이}] \quad \therefore y = c_1 + c_2x + A \cos x + B \sin x$$

4.12

$$[\text{풀이}] \quad \therefore y = c_1e^{3x} + c_2e^{-3x} + A \cos \sqrt{3}x + B \sin \sqrt{3}x$$

4.13

$$[\text{풀이}] \quad \therefore y = (A_1 + A_2x) \cos 2x + (B_1 + B_2x) \sin 2x$$

4.14

$$[\text{풀이}] \quad \therefore y = (A_1 + A_2x) \cos 2\pi x + (B_1 + B_2x) \sin 2\pi x$$

4.15

$$[\text{풀이}] \quad \therefore y = e^x + 3 \cos 10x + \sin 10x$$

4.16

$$[\text{풀이}] \quad \therefore y = \frac{1}{9}e^x - \frac{1}{3}\left(\frac{1}{3} + x\right)e^{-2x}$$

4.17

$$[\text{풀이}] \quad \therefore y = -\cos x + 2\sin x - x\cos x - \frac{1}{2}x\sin x$$

4.18

$$[\text{풀이}] \quad \therefore y = \frac{5}{4}e^{2x} + \frac{3}{4}e^{-2x} - 2\cos 2x + \frac{1}{2}\sin 2x$$

4.19

$$[\text{풀이}] \quad \therefore y = -1 - \frac{1}{2}e^x + \frac{1}{6}e^{-x} + \frac{1}{3}e^{2x}$$

4.20

$$[\text{풀이}] \quad \therefore y = y_h + y_p = c_1 + (c_2 + c_3x)e^{-x} + x^2$$

4.21

$$[\text{풀이}] \quad \therefore y = y_h + y_p = (c_1 + c_2x + c_3x^2)e^{-x} + 5x^3e^{-x}$$

4.22

$$[\text{풀이}] \quad \therefore y = c_1e^{5x} + c_2e^{-5x} + A\cos 5x + B\sin 5x - \frac{1}{16}\sinh x$$

4.23

$$[\text{풀이}] \quad \therefore y = c_1e^{1.5x} + c_2e^{-1.5x} + A\cos 2x + B\sin 2x - 0.014\sin \pi x + \frac{1}{14}e^{2x}$$

4.24

$$[\text{풀이}] \quad \therefore y = y_h + y_p = c_1 e^{2x} + c_2 e^{-2x} + A \cos 2x + B \sin 2x + \frac{1}{x}$$

4.25

$$[\text{풀이}] \quad \therefore y = 1 - \cos 2x + \sin 2x + \sin x$$

4.26

$$[\text{풀이}] \quad \therefore y = -\frac{1}{2}(e^x - e^{-x}) + \cos x + \sin x + 2(e^{2x} - e^{-2x})$$

4.27

$$[\text{풀이}] \quad \therefore y = 3xe^{-x} + x^2 - 6x + 12$$

4.28

$$[\text{풀이}] \quad \therefore y = 3e^{2x} - 4e^{-2x} + 2\cos x$$