

## [Chapter 05] 연습문제 정답

### 5.1

[풀이]

$$\lambda_1 = 1, \mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = -1, \mathbf{x}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

### 5.2

[풀이]

$$\lambda_1 = -1, \mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = -3, \mathbf{x}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

### 5.3

[풀이]

$$\lambda_1 = 0, \mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\lambda_2 = -3a, \mathbf{x}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

### 5.4

[풀이]

$$\lambda_1 = 0, \mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$\lambda_2 = -0.036, \mathbf{x}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

### 5.5

[풀이]

$$\lambda_1 = 4, \mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = -3, \mathbf{x}^{(2)} = \begin{bmatrix} 1 \\ -2.5 \end{bmatrix}$$

## 5.6

$$[\text{풀이}] \quad \therefore \mathbf{y} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t}$$

## 5.7

$$[\text{풀이}] \quad \therefore \mathbf{y} = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-5t} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{-t}$$

## 5.8

$$[\text{풀이}] \quad \therefore \mathbf{y} = c_1 \begin{bmatrix} -5 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{-\frac{1}{2}t}$$

## 5.9

$$[\text{풀이}] \quad \therefore \mathbf{y} = c_1 \begin{bmatrix} -3 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{9t}$$

## 5.10

$$[\text{풀이}] \quad \therefore \mathbf{y} = c_1 \begin{bmatrix} 0.75 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t$$

## 5.11

$$[\text{풀이}] \quad \begin{cases} y_1 = 4.5 \cos 2t - 0.8 \sin 2t \\ y_2 = -0.8 \cos 2t - 4.5 \sin 2t \end{cases}$$

## 5.12

$$[\text{풀이}] \quad \begin{cases} y_1 = 3 + 9e^{2t} \\ y_2 = -1 + 3e^{2t} \end{cases}$$

**5.13**

$$[\text{풀이}] \quad \begin{aligned} y_1 &= 2e^{3t} + \frac{1}{2}e^{-t} \\ y_2 &= 4e^{3t} - e^{-t} \end{aligned}$$

**5.14**

$$[\text{풀이}] \quad \begin{aligned} y_1 &= 2e^{3t} \\ y_2 &= 6e^{3t} \end{aligned}$$

**5.15**

$$[\text{풀이}] \quad \begin{aligned} y_1 &= 2e^{9t} - e^{3t} \\ y_2 &= 8e^{9t} - e^{3t} \end{aligned}$$

**5.16**

$$[\text{풀이}] \quad \therefore \mathbf{y} = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cos t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin t$$

**5.17**

$$[\text{풀이}] \quad \therefore \mathbf{y} = c_1 \begin{bmatrix} 1 \\ \sqrt{2}-1 \end{bmatrix} e^{\sqrt{2}t} + c_2 \begin{bmatrix} 1 \\ -\sqrt{2}-1 \end{bmatrix} e^{-\sqrt{2}t} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} e^t$$

**5.18**

$$[\text{풀이}] \quad \begin{aligned} y_1 &= c_1 e^{-9t} + c_2 e^{-3t} + 12t + \frac{92}{3} \\ y_2 &= 3c_1 e^{-9t} - 3c_2 e^{-4t} - 72t - 34 \end{aligned}$$

**5.19**

$$[\text{풀이}] \quad \therefore \mathbf{y} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ -4 \end{bmatrix} e^{3t} + \begin{bmatrix} 2t^2 - \frac{13}{3}t - \frac{101}{18} \\ 3t^2 - \frac{14}{3}t - \frac{271}{18} \end{bmatrix}$$

**5.20**

$$[\text{풀이}] \quad \therefore \quad \mathbf{y} = c_1 \begin{bmatrix} 4 \\ 3 \end{bmatrix} e^{10t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-4t} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \sin t$$

**5.21**

$$[\text{풀이}] \quad \begin{aligned} y_1 &= -4e^{-4t} - 8e^t + \frac{5}{2} + 13e^{2t} \\ y_2 &= -8e^{-4t} + 8e^t + 3 + 7e^{2t} \end{aligned}$$

**5.22**

$$[\text{풀이}] \quad \begin{aligned} y_1 &= -2\cos t + \sin t + t + 1 \\ y_2 &= 3\cos t + \sin t - t \end{aligned}$$

**5.23**

$$[\text{풀이}] \quad \mathbf{y} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} e^{2t} - 4 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t} - \begin{bmatrix} 7 \\ 3 \end{bmatrix} \cos t + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \sin t$$

**5.24**

$$[\text{풀이}] \quad \therefore \quad \mathbf{y} = -2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t - \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t} + \begin{bmatrix} -1 + 5e^{2t} \\ 3 - e^{2t} \end{bmatrix}$$

**5.25**

$$[\text{풀이}] \quad \therefore \quad \mathbf{y} = \frac{1}{20} \begin{bmatrix} -1 \\ 4 \end{bmatrix} e^{3t} - \frac{6}{5} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t} + \begin{bmatrix} \frac{5}{4} \\ 1 \end{bmatrix} e^{-t} + \begin{bmatrix} \frac{7}{6} \\ \frac{11}{6} \end{bmatrix} t + \begin{bmatrix} \frac{17}{36} \\ \frac{13}{36} \end{bmatrix}$$

**5.26**

$$[\text{풀이}] \quad \therefore \quad \begin{cases} I_1(t) = -4e^{-3t} + 4 \\ I_2(t) = 2e^{-3t} + 4 \end{cases}$$

5.27

$$\begin{aligned} [\text{풀이}] \quad \therefore \quad I_1 &= -\frac{200}{3}e^{-2t} + \frac{125}{3}e^{-0.8t} + 25 \\ I_2 &= -\frac{100}{3}e^{-2t} + \frac{100}{3}e^{-0.8t} \end{aligned}$$